T/S 1500 T/S 1000 ZX-81

Exploring Timex Sinclair 1500/1000 Graphics

Julius Guest

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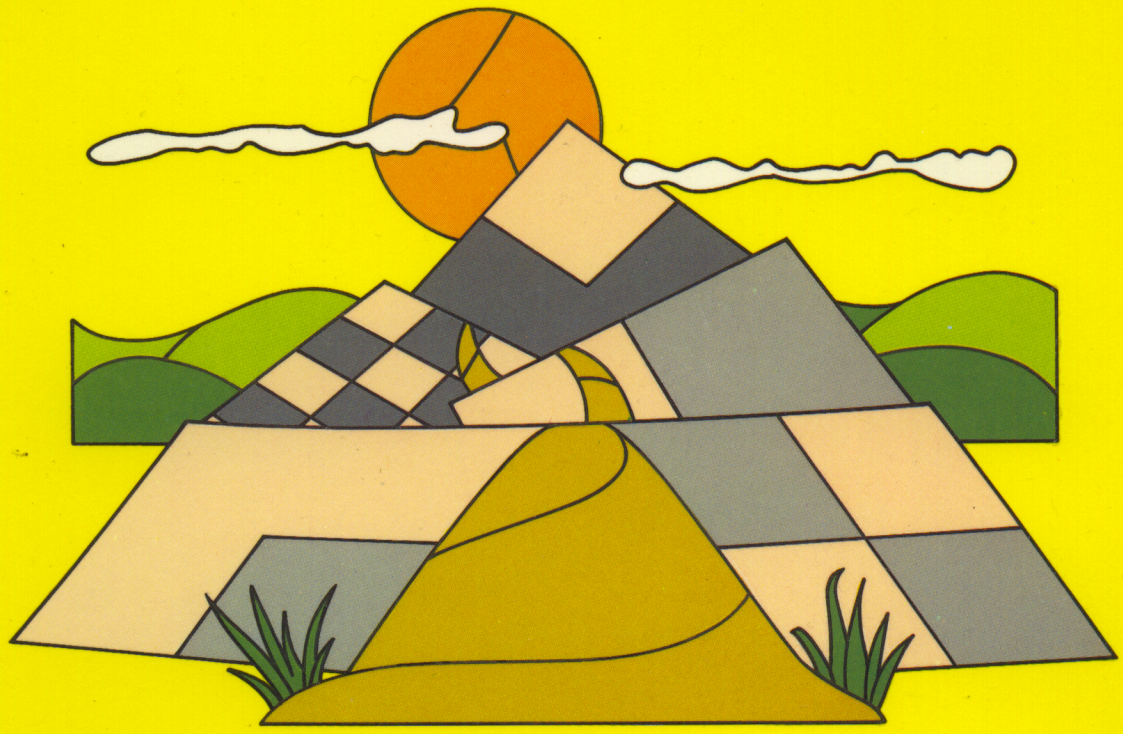

For the Timex Sinclair 1500, Timex Sinclair 1000 and ZX-81 Computers

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Julius Guest
May 1983

Introduction

This is a book for doing rather than just reading. This way you will have the double pleasure of both learning and, by running your own programs on the T/S 1000 or ZX81, seeing your designs on the T.V. screen.

The prerequisites are few. Of course you will need to have a T/S 1000 or ZX81 or easy access to one of these computers, and you will also need a black and white T.V. It would be helpful to have a small tape recorder for memory storage, to save yourself the trouble of retyping some of your successful programs. (One with a counter would be best.) For a few of the exercises in this book you will need the 16K RAM pack, and it could prove very useful if you want to do a number of more complicated designs.

Don't forget to keep the manual for your computer by your side as a good friend and ally. You are fortunate that this is a good manual and generally easy to follow. It would pay you to read the sections on BASIC in it before going any further. You will probably learn a fair amount of BASIC as you work through this book, but it is not designed to teach you BASIC programming as such. If you come across any BASIC commands you have not met before, look these up in the manual.

In the early chapters of this book many of the problems require very little mathematical background. However, some of the later problems assume that you are reasonably familiar with simple coordinate geometry. For those who need extra help here, the Mathematical Appendix gives further explanation. If any of the problems stump you, you will still be able to produce the graphics on your computer by using the programs listed in the solutions at the back of the book. There are many ways of solving these problems, so if your program works and differs from the one quoted here don't be surprised! It would be more surprising if they coincided.

Let us hope that the book will give as much pleasure to the user as it has to the author.

1 The Letter Grid

The letter grid consists of 22 horizontal lines (labelled from 0 to 21) and 32 vertical columns (labelled from 0 to 31). This is illustrated in Figure 1.1. The origin of this grid (i.e. the point of intersection of the two axes) is at the top left-hand corner of the screen. This makes good sense as we write from left to right and from top to bottom of the page.

To make sure we are correctly orientated to use the letter grid, let us place the letter X into the four corners of the grid. Enter the following commands:

```
PRINT AT 0,0;"X"
```

This places an X on the top left-hand corner of the screen.

```
PRINT AT 0,31;"X"
```

This places an X on the top right-hand corner.

```
PRINT AT 21,0;"X"
```

This places an X on the bottom left-hand corner.

```
PRINT AT 21,31;"X"
```

This places an X on the bottom right-hand corner.

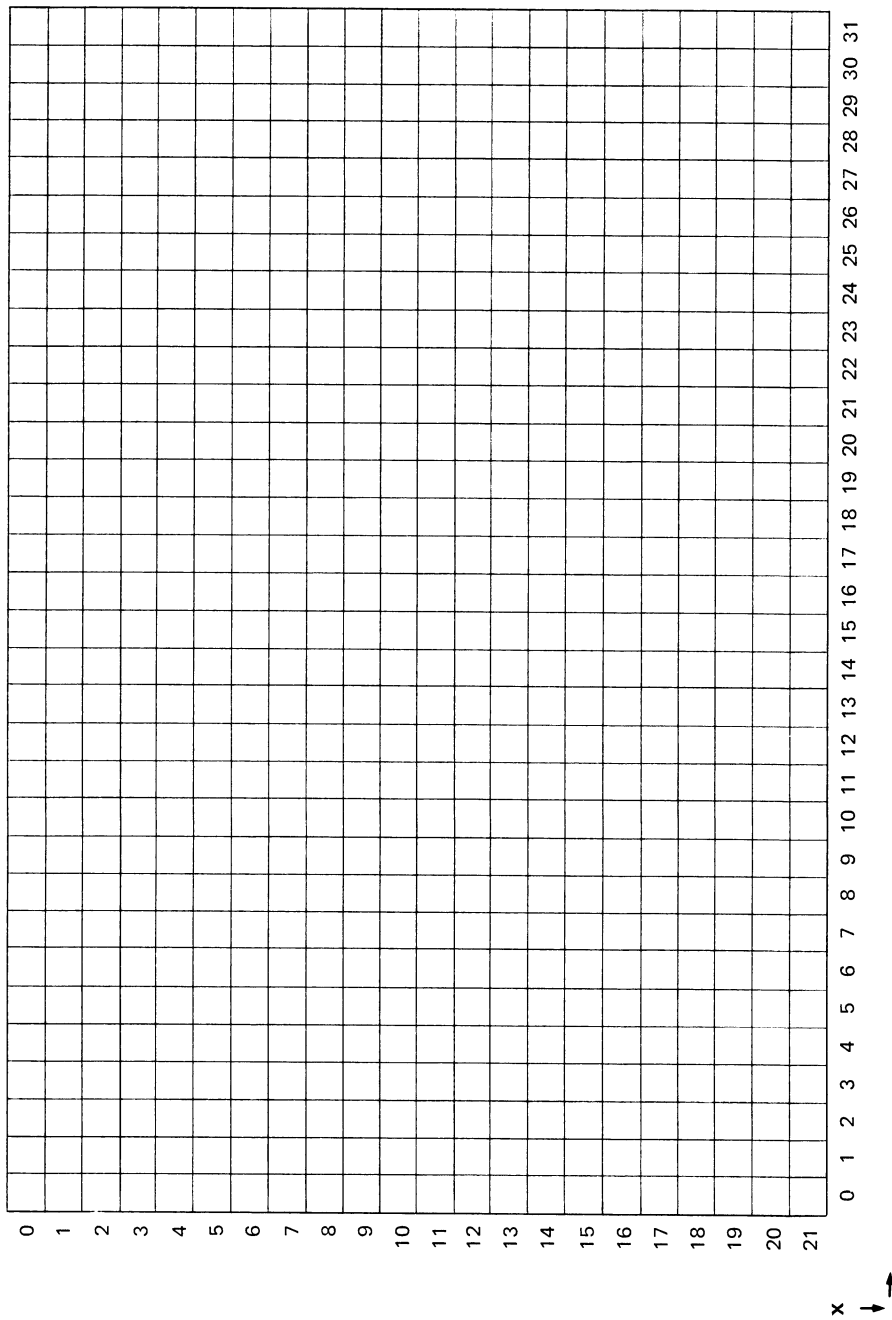


Figure 1.1 The Letter Grid

We could place any letter of the English alphabet into any one of the 704 squares on this grid. So if, for example, we wanted to place the letter O in the centre of the screen we would enter this command:

```
PRINT AT 11,16;"O"
```

Now do this.

Similarly, any of the figures from 0 to 9 may be placed in any square on the screen. But the computer allows more than this. The following 21 symbols may also be displayed:

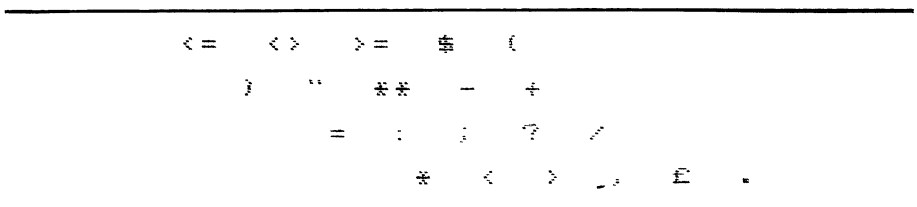


Figure 1.2

We can see these symbols on the screen by using the character set codes (abbreviated by the computer as CHR$). These are quoted on pages 181–7 (Appendix A) of your manual. We program as follows:

```
  10 PRINT AT 9,0;CHR$ 219;"  ";
CHR$ 221;"  ";CHR$ 220;"  ";CHR$
 13;"  ";CHR$ 15
  20 PRINT AT 11,3;CHR$ 17;"  ";
CHR$ 11;"  ";CHR$ 216;"  ";CHR$
22;"  ";CHR$ 21
  30 PRINT AT 13,7;CHR$ 20;"  ";
CHR$ 14;"  ";CHR$ 25;"  ";CHR$ 1
5;"  ";CHR$ 24
  40 PRINT AT 15,11;CHR$ 23;"  "
;CHR$ 19;"  ";CHR$ 18;"  ";CHR$
26;"  ";CHR$ 12;"  ";CHR$ 27
```

Check your results against Figure 1.2.

In addition to these symbols, there are some more which are particularly useful for our purposes. These are the 22 graphics symbols which are peculiar to this computer. They are clearly illustrated on page 78 (Chapter 11) of the manual. For example, let us place the symbol ▚ on the centre of the screen. This could be done in either of two ways:

Method 1: Type

```
PRINT AT 11,16;CHR$ 134
```

Method 2: Type

```
PRINT AT 11,16;"▝"
```

In this book Method 1 only is used because it is easier to type.

Here are some simple illustrations using the letter grid.

1. Write a program that forms a column by repeating the letter G on the screen. This may be done using an endless loop:

```
10 PRINT "G"
20 GOTO 10
```

2. Write a program that fills the entire screen by repeating the letter G. This is as easy as making a column; we simply add a semicolon to the PRINT command:

```
10 PRINT "G";
20 GOTO 10
```

The semicolon forces the computer to select the next position on the right.

3. What we can do with one letter could be done equally well with a word or a whole sentence. Try the following:

```
  10 PRINT "GRAPHING IS FUN ON T
HE ZX 81."
  20 GOTO 10
```

4. Write a simple program that forms a column made up of a random selection of the character codes 129 to 135.

```
10 LET A=INT (135*RND+1)
20 IF A<129 THEN GOTO 40
30 PRINT CHR$ A
40 GOTO 10
```

(Don't be surprised if you see no symbols on your screen for a while, as the ZX81 can't print anything until it reaches character codes 129 to 135.)

Problems

1. Write a program that forms a column made up of your initials (first and last initial only).

2. Fill up the whole screen with your two initials.

3. Write a program that forms two columns made up of your initials, one on the left side of the screen and the other on the right.

4. Suppose you have three initials. Write a program to form a column for your first initial on the left side of the screen, another column for your middle initial in the centre and a third column for your last initial on the right side of the screen.

5. The T/S 1000 or ZX81 also has "inverse video symbols", i.e. the symbol is shown in white on a black background. These are character codes 139 to 191, and they are listed in Appendix A of the manual. From this list we can see, for example, that inverse A is character code 166. Now fill up the whole screen with your initials in inverse video.

6. Fill the screen with a random selection of the 26 letters of the English alphabet. (Hint: Use character codes.)

7. Fill the screen with a random selection of the figures 0 to 9.

8. Fill the screen with a random selection of the figures 0 to 9 in inverse video.

9. Fill the screen with the following picture symbols in sequence, without leaving spaces between them:

10. Fill the screen with the symbols used above, but this time in random sequence.

11. Write a short program to inflict the "measles" on your poor screen, i.e. place a * sign at a number of randomly chosen points on it.

12. Write a program to list the 26 letters of the English alphabet in order, filling the entire screen.

13. List the 26 letters of the alphabet again, putting A in column 0, B in column 16, C in 0, D in 16, and so on.

14. Draw three equidistant vertical lines using only the character code 133.

15. Write a program to draw a giant multiplication sign on the screen. One diagonal starts at (0,0) and finishes at (21,21), whilst the other diagonal starts at (0,21) and ends at (21,0). Use an asterisk to draw the diagonals.

16. What modifications would you have to make in problem 15 to place your design in the centre of the screen? (See Figure 1.3.)

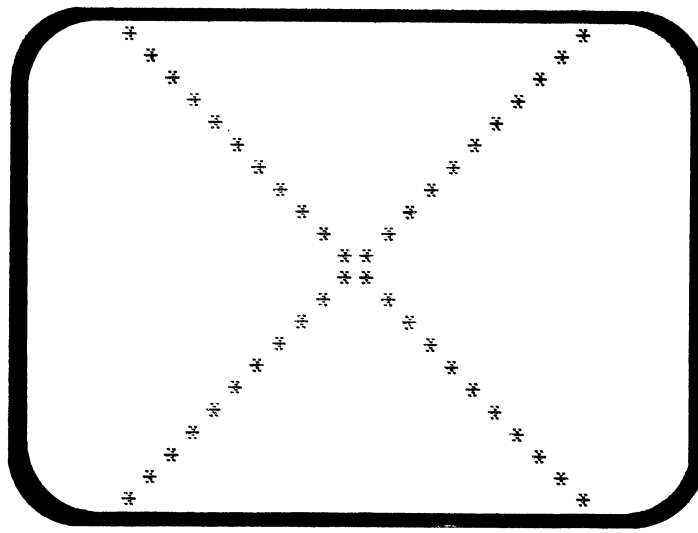


Figure 1.3

2 The Plot Grid

The plot grid is used for graphics and prints only a small black square called a pixel (short for picture element). This grid is divided into 44 horizontal lines (labelled from 0 to 43) and 64 vertical lines (labelled from 0 to 63). The origin is at the bottom left-hand corner of the screen with the x coordinate going horizontally from left to right and the y coordinate going vertically from bottom to top. (See Figure 2.1.) This grid contains $44 \times 64 = 2816$ unit squares, which is four times the number in the letter grid. Hence each element is a quarter of the size of a letter grid element.

To make sure we are correctly orientated to use the plot grid, let us place a pixel in each of the four corners of the screen.

Enter the following commands:

```
PLOT 0,43
```

This puts a pixel in the left-hand top corner of the screen.

```
PLOT 63,43
```

This puts a pixel in the right-hand top corner.

```
PLOT 0,0
```

This puts a pixel in the bottom left-hand corner.

```
PLOT 63,0
```

This puts a pixel in the bottom right-hand corner.

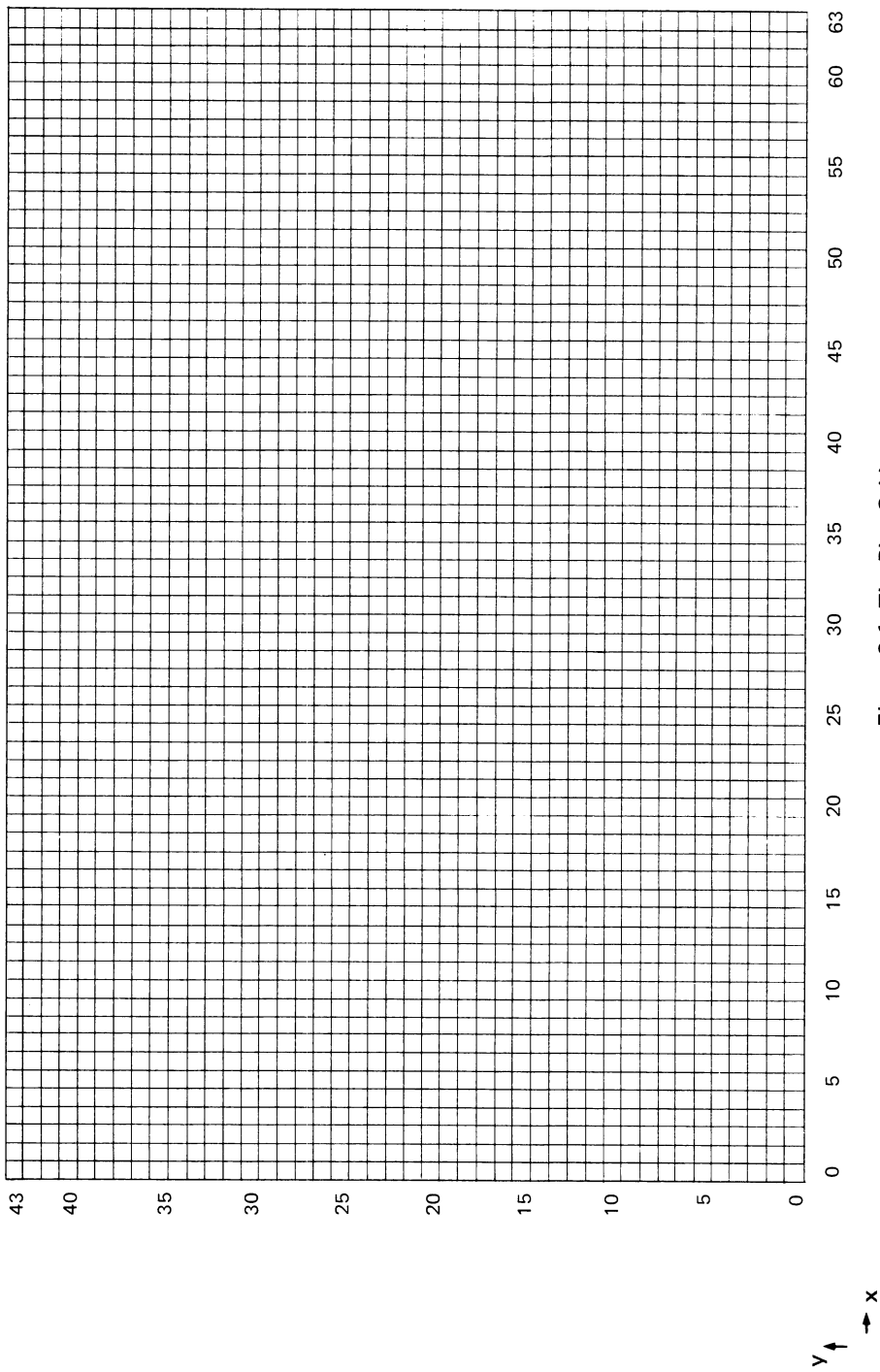


Figure 2.1 The Plot Grid

The opposite of the PLOT command is the UNPLOT command. By alternating these two commands it is possible to give the impression of movement.

Let's have a look at the following program:

```
10 LET K=0
20 PLOT K,22
30 UNPLOT K,22
40 LET K=K+1
50 IF K>63 THEN GOTO 61
60 GOTO 20
```

This program moves the pixel at (0,22) across the screen till it reaches (63,22). First we plot the pixel, then immediately erase it, then plot it again, then erase it again, and so on, giving the impression that it is moving across the screen. GOTO 61 takes us out of the program.

It is possible to use the PRINT and PLOT commands side by side. For example, we could enter the following program:

```
10 FOR K=0 TO 15
20 PLOT K,20
30 NEXT K
40 PRINT "HAPPY DAYS"
50 FOR K=35 TO 63
60 PLOT K,20
70 NEXT K
```

This program prints the message "HAPPY DAYS" with a horizontal line before the message and another horizontal line after it. Note that the PRINT command follows the last pixel of the left line and we do not need a PRINT AT command.

The main use of the plot grid is to draw simple mathematical functions. With the 16K RAM pack this usually presents no problem. However, without the RAM pack we frequently run out of memory space, so we can't draw the whole curve but only part of it. The 16K RAM pack gives the computer sixteen times as much memory.

Here are some simple illustrations using the plot grid.

1. Draw the curve $y = \sqrt{x}$ using the plot grid.

In order to see as much of this curve as the screen can show, it is expedient to use a scaling factor, i.e. let us select a such that

$$y = a\sqrt{x}$$

and find a when $x = 63$, $y = 43$ at the top right of the screen.
Thus

$$a = \frac{43}{\sqrt{63}}$$
$$= 5.4175$$

We enter the following program to draw the curve:

```
10 FOR X=0 TO 63 STEP 6
20 PLOT X,5.4175*SQR (X)
30 NEXT X
```

The reason for using STEP 6 in line 10 is that this gives a fair approximation to the shape of the curve. This is the best that can be done with the resolution at our disposal. For the meaning of STEP 6, see page 84 (Chapter 12) of the manual. It might be wise to run the program without STEP 6 and note the difference.

2. Draw the curve $y=e^{-x}$ for $x>0$.
Here two scaling factors are needed. To get the right scaling let

$$y = ae^{-bx}$$

and fix a and b such that for $x=0$, $y=43$ and the slope of the curve at that point is -1.
Thus $a=43$ and $b=1/43$. So we program as follows:

```
10 FOR X=0 TO 63 STEP 6
20 PLOT X,43*EXP (-X/43)
30 NEXT X
```

3. Draw the curve $y=\arcsin x$ with its origin at the centre of the screen.
As the origin is at (32,22) let

$$y=22+ a \arcsin \frac{(x-32)}{b}$$

This will ensure that for $x=32$, $y=22$.
For convenience let us place the curve between $x=10$ and $x=54$ in the plot grid.
To fix a and b when $x=54$:

$$\frac{x-32}{b}=1$$

Hence $b=22$.
For $x=54$, y should equal 43.

Hence $43=22+a\left(\frac{\pi}{2}\right)$ and therefore $a=42/\pi$.

We may now program as follows:

```
10 FOR X=10 TO 54 STEP 3
20 LET Y=X-32
30 PLOT X,22+42/PI*ASN (Y/22)
40 NEXT X
```

4. It is often desirable to draw up a function table to find the general shape of a curve. The curve is then drawn on a piece of graph paper. Then we must transfer it onto the screen.

Write a small program to draw up a function table for $y=x(x^2-1)$ for values of x lying between -4 and $+4$ in steps of 0.5. Then write a program to plot this curve suitably scaled on the screen.

The function table will give us an indication whether our y values will over- or undershoot the grid as we move from left to right.

Enter the program as follows:

```
10 PRINT "ΔΔXΔΔΔΔΔΔΔΔΔΔΔΔΔΔY"
20 PRINT ;;
30 FOR X=-4 TO 4 STEP 0.5
40 LET Y=X*(X*X-1)
50 PRINT "ΔΔ";X;"ΔΔΔΔΔΔΔΔΔΔ";Y
60 NEXT X
```

When we run this program we can see that y does indeed exceed the edges of the grid. However, we can see also that the centre of the screen would be a good origin for the curve. This is the solution to the scaling problem. Thus we program as follows:

```
  10 FOR X=-1.5 TO 1.5 STEP 0.04
6875
  20 LET Z=32+21.333*X
  30 LET Y=11.73*X*(X*X-1)
  40 PLOT Z,22+Y
  50 NEXT X
```

Lines 10 and 20 ensure that we get 64 points across the screen whilst line 30 takes care of a sensible scaling for y.

5. Set up a program to tabulate the function

$$y = e^{-x} \sin x \text{ for } x \text{ in } [0,4]$$

in steps of 0.5 and then write a program to plot this curve with a suitable origin and scale on the screen.

For the function tabulation we use the same program as in (4) above except that now lines 30 and 40 read:

```
30 FOR X=0 TO 4 STEP 0.5
40 LET Y=EXP (-X)*SIN X
```

Looking at the size and sign of y, we recognize that the y values should be increased by a factor of 50. So the relevant program is:

```
10 FOR X=0 TO 4 STEP 0.0625
20 LET Z=15.75*X
30 LET Y=22+50*EXP (-X)*SIN X
40 PLOT Z,Y
50 NEXT X
```

Lines 10 and 20 ensure that x extends across the whole screen, whilst line 30 takes care of the y scaling.

Problems

1. Write a short program to let a "pixel frog" move from right to left across the screen.

2. Write a short program to let the pixel frog move diagonally from (0,0) across to (63,43) of the plot grid and then return to its starting point.

3. It's lunchtime for the pixel frog and it is hungry. It knows there is a fly at the centre of the screen (i.e. $x=32, y=22$ of the plot grid), but the only way to reach it is to run in straight lines from (0,0) up to (0,43), then down along (1,43) to (1,0), then up again, and so on. Write a program to enable the pixel frog to reach the fly.

4. How would you modify your program to let the pixel frog run twice as fast?

5. Suppose the screen represents the target of a shooting gallery. Write a program to simulate 50 random shots. (Each shot is shown as a pixel point.)

6. We are still at a shooting gallery and you are allowed 50 random shots as before. But this time delete each shot after it has been fired and stop shooting if your shot lands inside [30,34] for the x coordinate and inside [20,24] for the y coordinate, i.e. the bull's-eye area.

7. Write a program to let the pixel frog run in a straight line from (0,0) up to (0,43) and from there run horizontally across the screen to (63,43).

8. Write a simple program to draw a function table for $y=x^2-20x+105$ for x lying between 0 and 16 in equal intervals of 1.

9. Using the information from problem 8 write a short program to draw the curve $y=x^2-20x+105$ for suitable values of x and y in equal intervals of 1.

10. Write a simple program to draw a function table for

$$y = \frac{40}{x^2+1}$$

for x lying between -10 and $+10$ in steps of 1.

11. Using the information gathered from problem 10 above, write a program to plot the curve

$$y = \frac{40x}{x^2 + 1}$$

with suitable scaling. (Hint: A useful origin is (32,0) here.)

12. (a) Write out a simple program to tabulate

$$y = \frac{60x}{x^2 + 1} \text{ for } x \text{ in } [0,60]$$

in steps of 1.

(b) Hence write a program to plot the curve

$$y = \frac{60x}{x^2 + 1} \text{ for } x \geqslant 0$$

using suitable scaling.

(c) Write a suitable plot program for the given curve when x may have either sign. (Hint: Make the origin at the centre of screen.)

If you have the 16K RAM pack try the following:

13. Remember problem 3 above where the pixel frog started at (0,0) and found the fly at the centre of the screen? Well, had it gone on it would have run out of memory. Let's now use our 16K RAM pack and let the frog run all the way across the screen to collect the fly finally at (63,0). Make the necessary modifications to the program to let the pixel frog do this.

14. In problem 7 the pixel frog runs along two sides of the screen. Had it gone on it would have again run out of memory. Using the 16K RAM pack let the pixel frog complete a whole lap, i.e. go right round the screen.

3 Drawing Lines

Horizontal and vertical straight lines may be drawn using either the letter grid or the plot grid.

For example, to draw a horizontal straight line across the centre of the screen using the letter grid, we would program as follows:

```
10 FOR K=0 TO 31
20 PRINT AT 11,K;CHR$ 128;
30 NEXT K
```

This method produces a thick line, as we should expect. To draw a thinner line, we use the plot grid. The corresponding program using the plot grid is:

```
10 FOR K=0 TO 63
20 PLOT K,22
30 NEXT K
```

Now you devise the relevant two programs to generate a vertical straight line down the centre of the screen.

Drawing a number of parallel horizontal straight lines is not much harder than drawing one. This will of course involve a double FOR loop. Using the letter grid, we would program as follows for generating, say, five such lines:

```
10 FOR R=0 TO 20 STEP 5
20 FOR K=0 TO 31
30 PRINT AT R,K;CHR$ 128;
40 NEXT K
50 NEXT R
```

Using the plot grid we would proceed in this way:

```
10 FOR R=0 TO 40 STEP 10
20 FOR K=0 TO 63
30 PLOT K,R
40 NEXT K
50 NEXT R
```

Now you devise the relevant two programs to draw five vertical parallel straight lines.

To draw oblique straight lines we generally use the plot grid unless the slope of the lines is either $+1$ or -1. In that case, we could use the character set code 6 for a $+1$ slope and character set code 134 for a -1 slope. To illustrate, let us draw the straight line with slope $+1$ starting at (18,5) and finishing at (2,21).

```
10 FOR Y=5 TO 21
20 PRINT AT 23-Y,Y;CHR$ 6
30 NEXT Y
```

Now you write the program for linking (1,1) to (21,21) with a line of slope -1.

For drawing oblique straight lines with a slope other than $+1$ or -1, two methods are open to us. Method A uses the straight line equation $y=a+bx$, whilst method B uses the form $x=c+t, y=d+kt$ for suitable t and given point P (c,d) and slope k. (For an explanation of the mathematics here, see the Mathematical Appendix.)

To illustrate method A let us draw the line joining (3,2) to (57,29). Using these values of x and y, we have two equations to solve:

$a+3b=2$ and
$a+57b=29$

Hence $a=\frac{1}{2}$ and $b=\frac{1}{2}$. We program as follows:

```
10 FOR X=3 TO 57 STEP 2
20 PLOT X,(1+X)/2
30 NEXT X
```

You will note that the line is not continuous but broken into small segments. This is the best we can do on our computer (and many others, for

that matter) as the resolution is not fine enough. If we insist on continuity, and replace line 10 by

```
10 FOR X=3 TO 57
```

we obtain a staircase and the image of our straight line is lost.

Now you write a program to draw the same straight line, this time using method B.

Let us conclude this chapter with a few illustrations.

1. Using the plot grid, divide the screen into nine rectangles by drawing two parallel vertical straight lines and two parallel horizontal straight lines. (See Figure 3.1.)

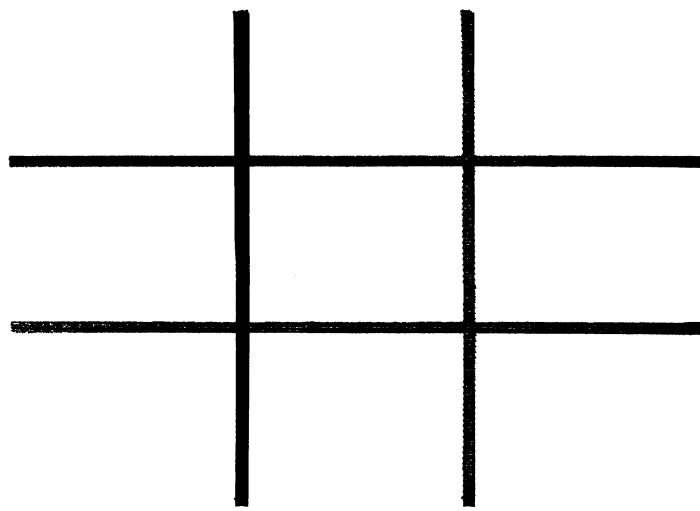


Figure 3.1

The program

```
10 FOR Y=15 TO 30 STEP 15
20 FOR X=0 TO 63
30 PLOT X,Y
40 NEXT X
50 NEXT Y
```

will produce the two horizontal lines whilst

```
 60 FOR X=21 TO 42 STEP 21
 70 FOR Y=0 TO 43
 80 PLOT X,Y
 90 NEXT Y
100 NEXT X
```

will produce the two vertical lines.

2. For the following illustration, you will need the 16K RAM pack. Write a program to draw a line along the four boundaries of your screen and then draw in the two diagonal lines from corner to corner. (See Figure 3.2.)

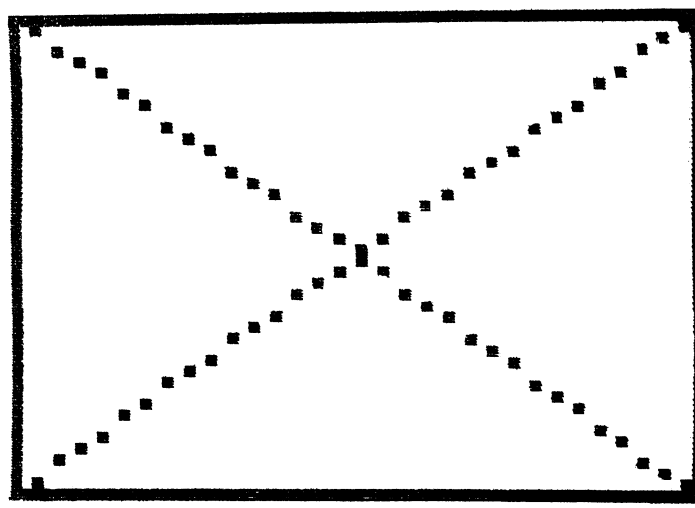


Figure 3.2

First enter the following program:

```
 10 FOR Y=0 TO 43 STEP 43
 20 FOR X=0 TO 63
 30 PLOT X,Y
 40 NEXT X
 50 NEXT Y
 60 FOR X=0 TO 63 STEP 63
 70 FOR Y=0 TO 43
 80 PLOT X,Y
 90 NEXT Y
100 NEXT X
```

This produces the straight lines along the four boundaries. For the two diagonal lines write:

```
110 FOR X=0 TO 63 STEP 2
120 PLOT X,43*X/63
130 NEXT X
140 FOR X=0 TO 63 STEP 2
150 PLOT X,-43*(X-63)/63
160 NEXT X
```

That's it! You have a giant envelope.

There are two more points to be made about this exercise. First, if you try this program without the 16K RAM pack you won't even be able to finish all four boundaries. Second, if the following program is used for the two diagonals, only one FOR loop is needed:

```
110 FOR X=0 TO 63 STEP 2
120 PLOT X,43*X/63
130 PLOT X,-43*(X-63)/63
140 NEXT X
```

3. Write a program to cover the screen with seven equidistant parallel straight lines of the type $y=4x+a$.

For example, if line 1 starts at (0,0) it would finish at (10·75,43); if line 2 starts at (8,0) it would finish at (18·75,43); and so on.

Program as follows:

```
10 FOR A=0 TO 192 STEP 32
20 FOR Y=0 TO 43 STEP 2
30 PLOT (Y+A)/4,Y
40 NEXT Y
50 NEXT A
```

4. If you have a 16K RAM pack you may use the program in illustration 3 above, with an additional program, to draw a further seven equidistant parallel straight lines of the type $y=-4x+b$. The first line starts at (43,0) and finishes at (10·5,0), and so on.

To the program above add:

```
 60 FOR B=43 TO 235 STEP 32
 70 FOR Y=43 TO 0 STEP -2
 80 PLOT (B-Y)/4,Y
 90 NEXT Y
100 NEXT B
```

5. Write a suitable program to form on the screen two mountains made up of four straight-line segments. (See Figure 3.3.) For this you also need the 16K RAM pack.

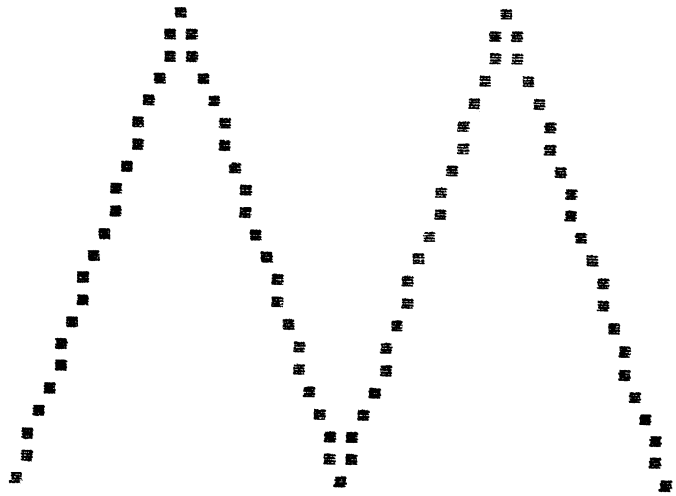


Figure 3.3

Begin by drawing the two parallel straight lines that have positive slope:

```
10 FOR A=0 TO -30 STEP -30
20 FOR Y=0 TO 42 STEP 2
30 PLOT (5*Y-14*A)/14,Y
40 NEXT Y
50 NEXT A
```

Now draw the two parallel straight lines that have negative slope:

```
 60 FOR B=30 TO 60 STEP 30
 70 FOR Y=0 TO 42 STEP 2
 80 PLOT (14*B-5*Y)/14,Y
 90 NEXT Y
100 NEXT B
```

6. Place a smaller mountain with parabolic shape on the two mountains of the previous illustration. (See Figure 3.4; also see the Mathematical Appendix if you require more detailed explanation here.)

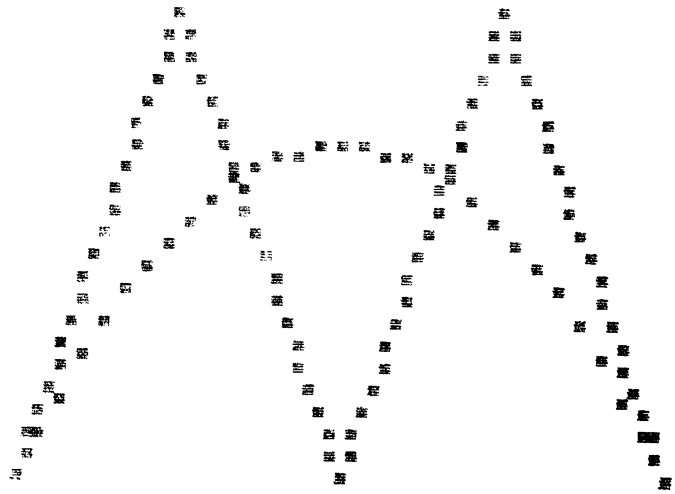


Figure 3.4

As the parabola goes through the origin, assume $y=ax^2+bx$. Now solve the equation for $x=30$, $y=30$ and for $x=60$, $y=0$.

Hence:

$$900a + 30b = 30$$
$$3600a + 60b = 0$$
$$a = -1/30,\ b = 2.$$

Therefore the program we require begins with lines 10 to 100 above, and continues:

```
110 FOR X=0 TO 60 STEP 2
120 PLOT X,2*X-X*X/30
130 NEXT X
```

Problems

1. Using the plot grid, divide your screen into nine rectangles by drawing two dotted horizontal straight lines and two dotted vertical straight lines.

2. Repeat problem 1 using the letter grid. (See Figure 3.5.)

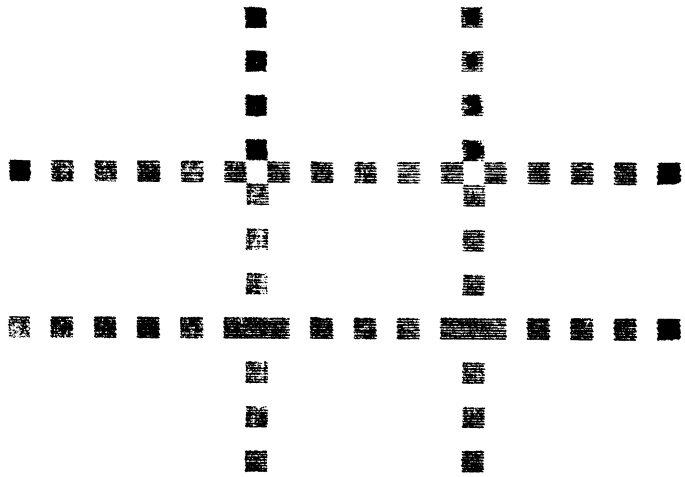


Figure 3.5

3. Write a program to join the three points A (2,5), B (22,30), and C (52,5) by two straight lines.

4. Write a program to join the four points A (1,1), B (16,31), C (41,6), and D (59,42) by three straight lines.

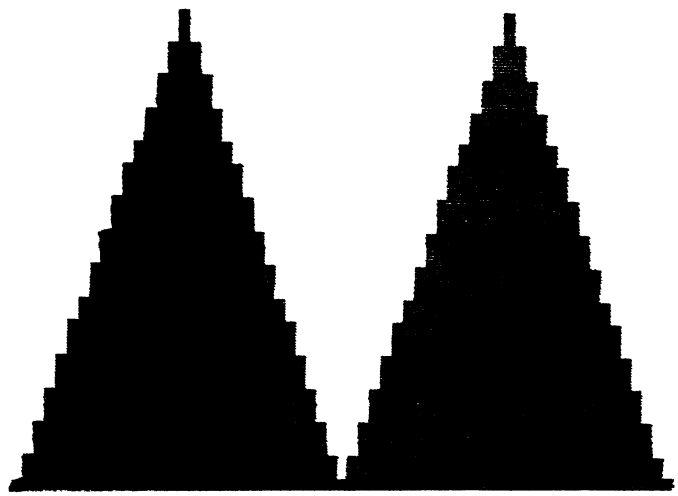


Figure 3.6

5. Write a program to form a large W on the screen. (You will need the 16K RAM Pack.)

6. Superimpose the letter M on your W.

7. Write a program to fill in the two mountains drawn in illustration 5 on page 21. (See Figure 3.6.)

8. Write a program to fill in the parabolic mountain as drawn in illustration 6 on page 22. (See Figure 3.7.)

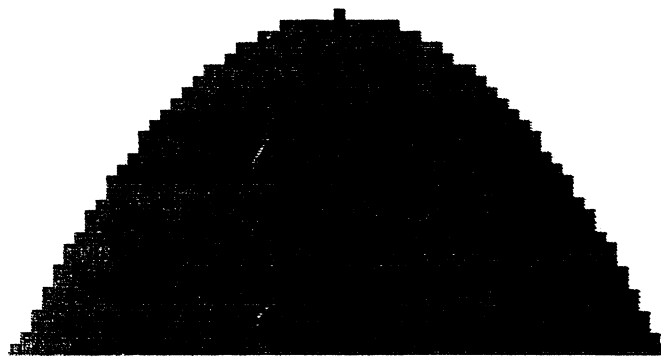


Figure 3.7

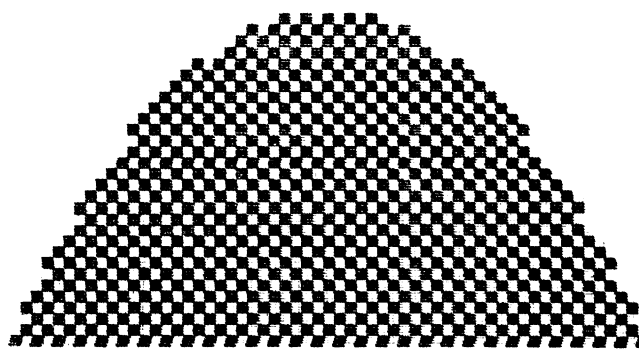


Figure 3.8

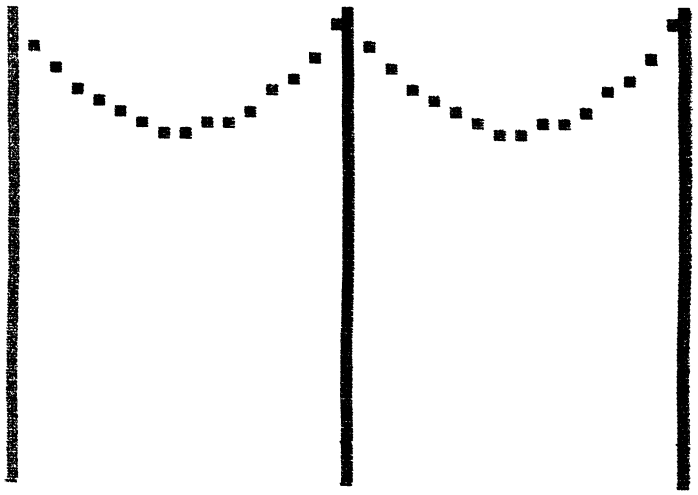


Figure 3.9

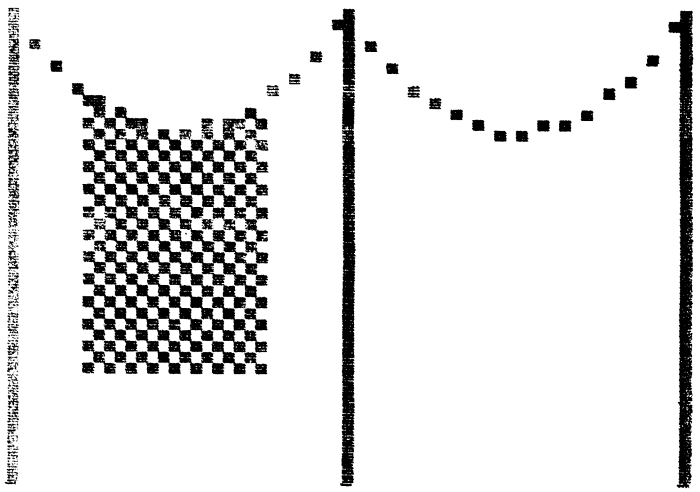


Figure 3.10

9. Fill in the parabolic mountain of the previous problem in checkerboard fashion. (See Figure 3.8.)

10. Write a program for drawing a washing line. (See Figure 3.9.)

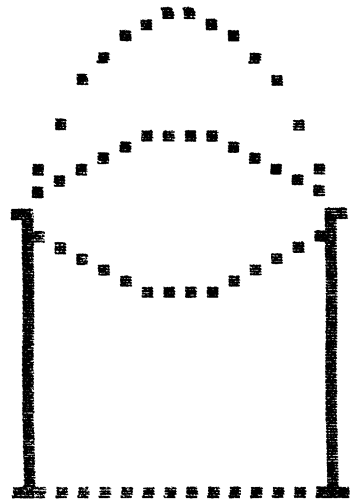


Figure 3.11

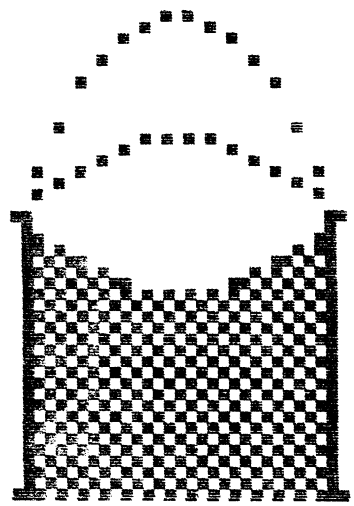


Figure 3.12

11. Write a program to hang one checkered sheet on the washing line. (See Figure 3.10.)

12. Write a program to draw a shopping basket. (See Figure 3.11.)

13. Put a checkerboard pattern on the shopping basket. (See Figure 3.12.)

4 Playing Tic Tac Toe (alias Noughts and Crosses)

To play Tic Tac Toe we must first set up the field. If we have no extra memory we cannot afford to draw the basic two horizontal and two vertical lines. So we do the next best thing, which is to define each of the nine squares by their corner points. This is easily done by using a double FOR loop:

```
10 FOR Y=0 TO 30 STEP 10
20 FOR X=0 TO 21 STEP 7
30 PRINT AT X,Y;"+";
40 NEXT X
50 NEXT Y
```

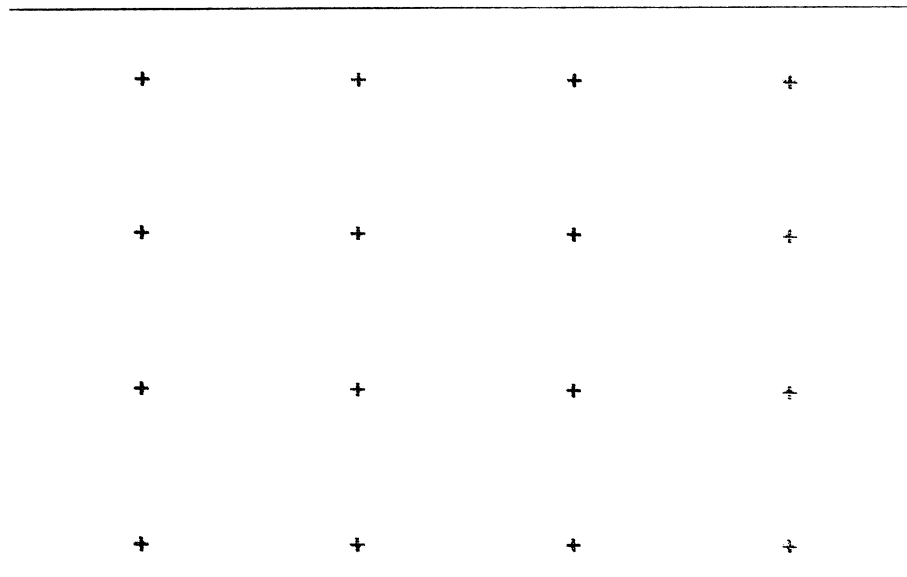


Figure 4.1

The field is now marked as shown in Figure 4.1. Now you find nine suitable sets of coordinates giving the positions for either X or 0 in any of the nine squares.

Let us assume that the first player always uses an X. The computer must now be told where he wants to place his X. This is done with the INPUT command:

```
60 INPUT X1
70 INPUT Y1
80 PRINT AT X1,Y1;"X"
```

To enter the 0 for the second player we program similarly:

```
 90 INPUT X2
100 INPUT Y2
110 PRINT AT X2,Y2;"O"
```

To return the play to the first player, we use a simple GOTO command. You find it! Then the game can begin.

When we finish one game and wish either to play another or do something else, we type 30 ENTER and 40 ENTER which results in the error condition B/80. If we then type ENTER again the Tic Tac Toe program reappears, and typing RUN and ENTER will start the game again. (On some machines the command is NEWLINE instead of ENTER.) In general we may stop any running program by typing BREAK.

With the extra memory provided by the 16K RAM pack, the Tic Tac Toe field is set up as follows:

```
 10 FOR Y=0 TO 30 STEP 10
 20 FOR X=0 TO 21
 30 PRINT AT X,Y;CHR$ 128
 40 NEXT X
 50 NEXT Y
 60 FOR X=0 TO 21 STEP 7
 70 FOR Y=0 TO 30
 80 PRINT AT X,Y;CHR$ 128
 90 NEXT Y
100 NEXT X
```

Problems

Although the pixel frog is not smart enough to play Tic Tac Toe, it can jump cheerfully from one square to the next.

1. Write a program to make it jump from the left-hand bottom corner square to the left-hand top corner square and from there horizontally across the screen to the right-hand top corner square. (Allow about one second for each jump.)

2. Devise a program to let the pixel frog jump along all the boundary squares and let it finish where it started.

3. Devise a program to let the pixel frog jump up all the left-hand squares, starting from the left-hand bottom corner square, then down all the centre squares and finally up all the right-hand squares.

5 Drawing A Chessboard

The basic element in a chessboard is, of course, the square — the board is itself a square with eight small squares along each side. Before we set up the board, we must decide what size the small squares will be on our grid. We don't want them too small, so we will use the letter grid. Let us try a 3 unit × 3 unit square. For this, we need 24 units along both sides of the grid. The vertical x coordinate on the letter grid runs from 0 to 21 only, so a 3 × 3 square is too big. We must use a 2 × 2 square.

Now there are many ways in which we could draw this square. One possibility is to program:

```
1000 PRINT AT X,Y;A$;A$;
1010 PRINT AT X+1,Y;A$;A$
```

where (x, y) are the top left-hand corner coordinates of the square and A$ stands for CHR$ 128, which is a black square of one unit length. As we shall use 32 of these 2 × 2 squares it is best to use a subroutine to avoid needless repetitions. To begin with we could draw the 16 black squares shown in Figure 5.1 as follows:

```
  5 LET A$=CHR$ 128
 10 FOR X=0 TO 12 STEP 4
 20 FOR Y=0 TO 12 STEP 4
 30 GOSUB 100
 40 NEXT Y
 50 NEXT X
      .
      .
      .
100 FOR K=0 TO 1
110 PRINT AT X+K,Y;A$;A$;
120 NEXT K
130 RETURN
```

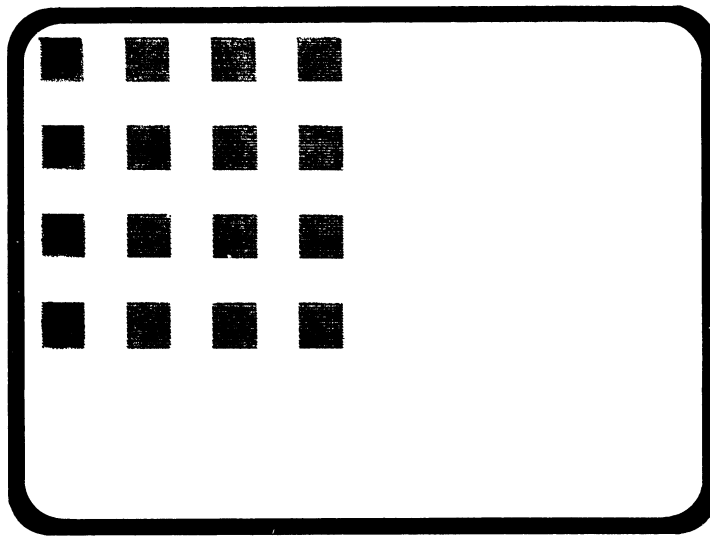


Figure 5.1

Now you work out a similar double FOR loop to fill in the other 16 black squares which we need to complete the chessboard (Figure 5.2).

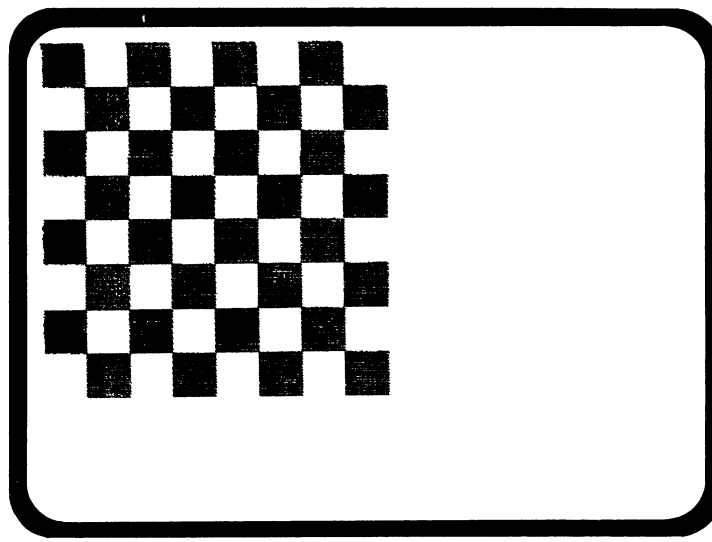


Figure 5.2

Let us now place the design in the centre of the screen, as in Figure 5.3. How would we have to modify lines 10, 20, 60 and 70 to do this?

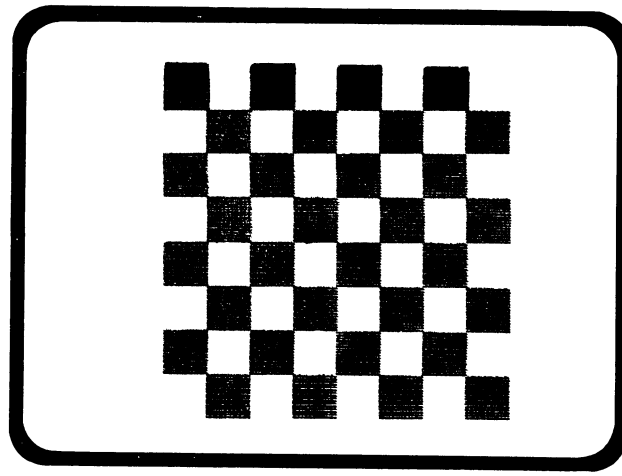


Figure 5.3

Problems

1. Write a program to obtain the 4 × 4 chessboard pattern as shown in Figure 5.4.

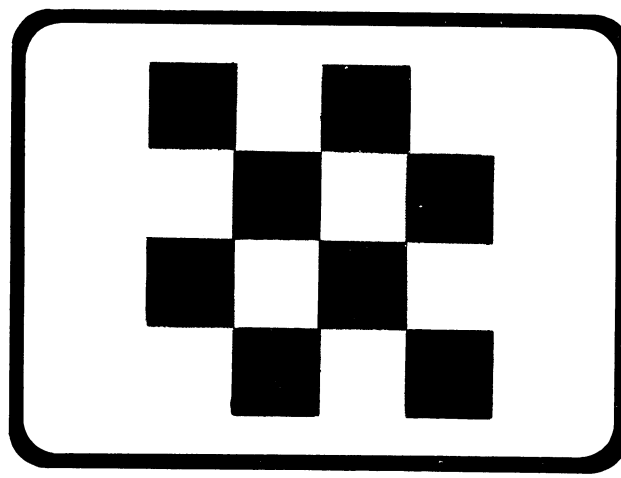


Figure 5.4

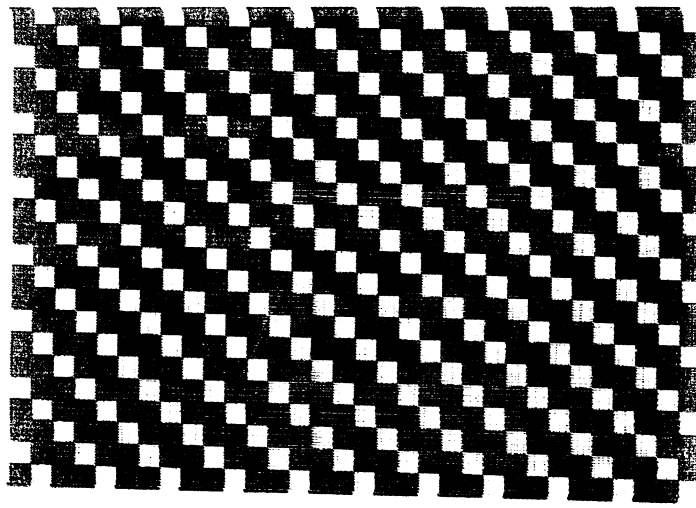


Figure 5.5

2. Replace CHR$ 128 by any of CHR$ 8, CHR$ 6, CHR$ 134, CHR$ 136, CHR$ 137, CHR$ 138 and keep the one which pleases you most.

3. Can you work out a *short* program to make the design shown in Figure 5.5?

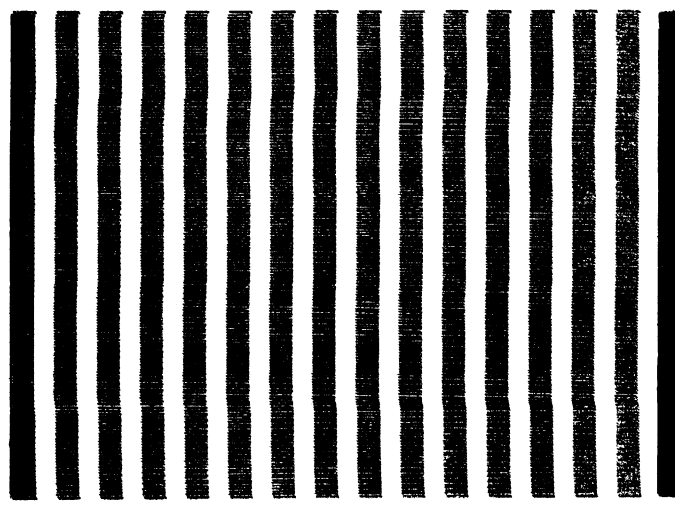


Figure 5.6

4. Can you make up a short program to fill your screen with 16 vertical straight lines? (See Figure 5.6.)

For the remaining three problems you will need the 16K RAM pack. The pixel frog is back again and although it can't play chess it enjoys jumping on any white square of a chessboard.

5. Write a program to let the pixel frog jump from the bottom left-hand corner of the chessboard diagonally across to the top right-hand corner. (Allow about one second for each jump.)

6. The pixel frog now decides to reach the top right-hand corner square by a different route. It starts out as before from the left-hand bottom corner square and now proposes to jump on all the white squares on the left boundary of the board, then move horizontally along the top row until it reaches its destination.

7. As a tour de force, the pixel frog now plans to jump on every white square of the chessboard. Write a suitable program to let it achieve its ambition.

6 Treasure Hunt

To play this game, some treasure must be hidden in a particular spot on the screen. The object is to locate it in the least number of moves. We guess where the treasure might be, then the computer tells us whether it is hidden to the left or right of where we have guessed, or higher or lower on the screen. We can use either grid, so let us use the plot grid (see Figure 6.1) to make the game more interesting. Remember that the plot grid extends horizontally from $x=0$ across to $x=63$, and vertically from $y=0$ across to $y=43$. The origin is at the left-hand bottom corner of the screen. Now there is little point in hiding a treasure ourselves and then hunting for it, because we would know already where we had placed it on the screen. So let us ask the computer to hide it.

This is done quite easily on setting

```
1000 LET A=INT (63*RND)
1010 LET B=INT (43*RND)
```

The treasure is now located at (A,B) and only the computer knows its exact location.

Before we continue, let us make quite sure that this works. Suppose we program:

```
10 LET A=INT (63*RND)
20 LET B=INT (43*RND)
30 PRINT "(";A;",";B;")";
```

and run it a few times. Do this and you will find that the x coordinate will always lie within [0,63] and the y coordinate will always lie within [0,43].

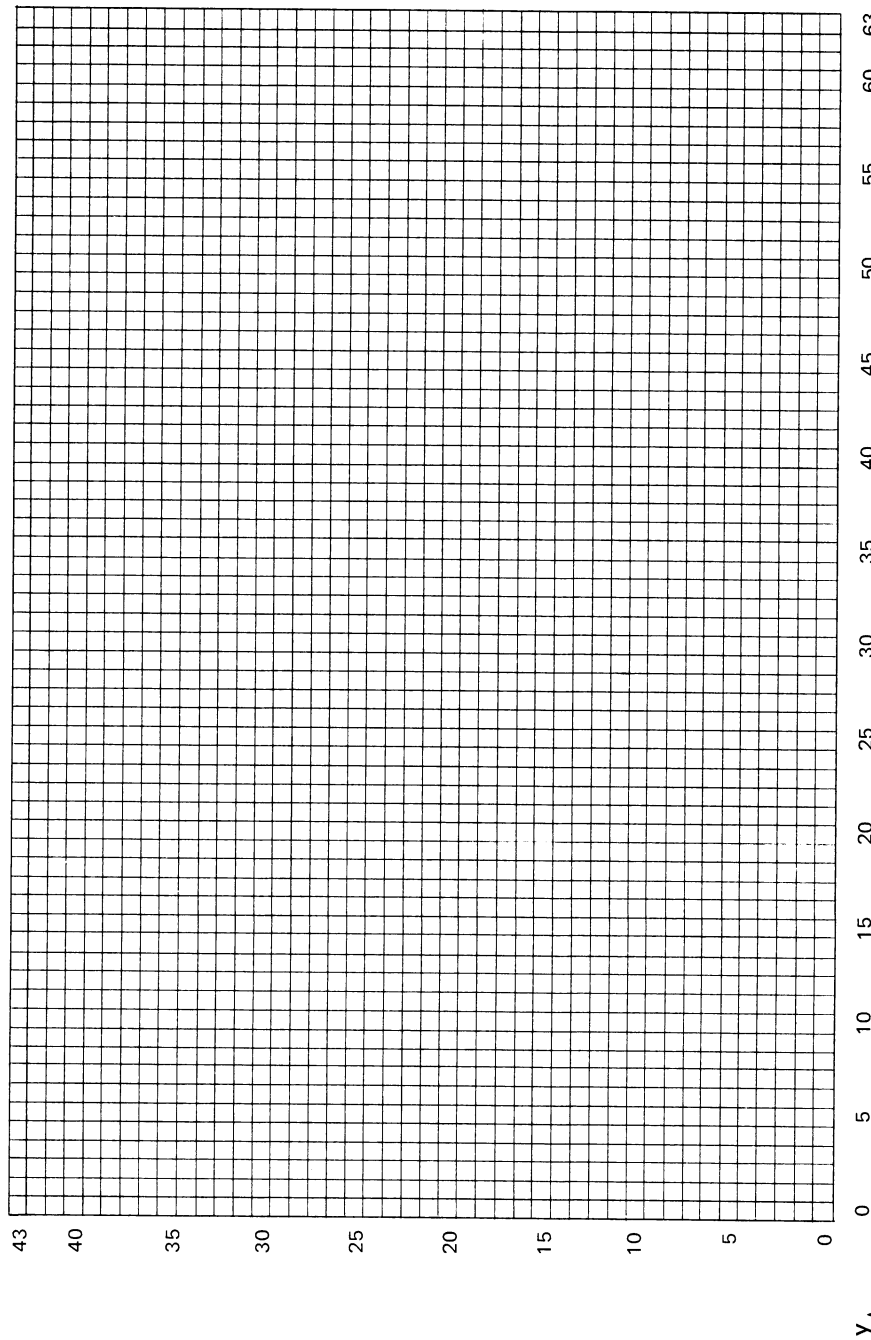


Figure 6.1 The Plot Grid

The next step, if we wish to play this game seriously, is to make a copy of the plot grid as shown in Figure 6.1. (Once you have drawn it on a sheet of paper, photocopy it several times to use in later games.) The copy of the plot grid enables us to keep a record of the moves that we have made, and thus forms part of the strategy for the game.

The hunt begins with the player selecting a point on the screen. This can be done with the two input commands:

```
30 INPUT X
40 INPUT Y
```

Next we need to ask the computer whether the x is too far left or right. This may be programmed as follows:

```
 50 IF X>A THEN GOTO 200
 60 IF X<A THEN GOTO 250
 70 PRINT "YOUR X IS O.K."
      .
      .
      .
      .
200 PRINT "TREASURE IS LEFT."
210 GOTO 80
250 PRINT "TREASURE IS RIGHT."
260 GOTO 80
```

Now you write the corresponding program segment for y. (When you do this don't forget to include a message to tell the player that he has found the treasure if he has guessed both x and y coordinates correctly.) That's all. You can now play.

By the way, if you wish to stop your search before you have found the treasure, type in STOP or BREAK.

Problems

1. How would you modify your program to hunt the treasure in a simple 5×5 square? (See Figure 6.2.)

2. How would you modify your program to count the number of moves you make?

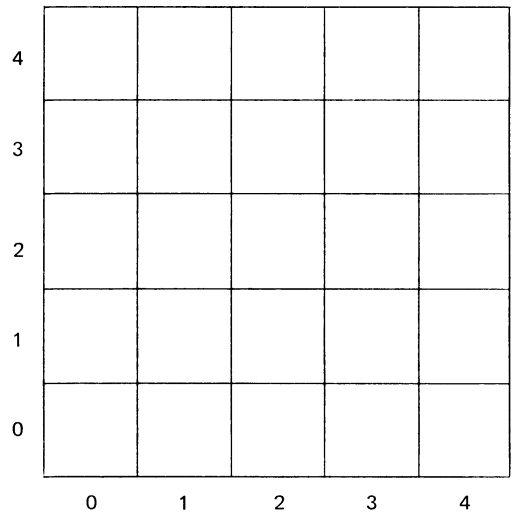


Figure 6.2

3. How would you modify the treasure hunt game to play it on the letter grid? (Remember that the origin in the letter grid is at the top left-hand corner of the screen and that *x* runs vertically and *y* horizontally.)

4. What program steps would you need to add to show the treasure on the screen?

7 Drawing Simple Polygons

The simplest polygon is the triangle. Suppose we wish to draw a triangle with vertices A (10,10), B (50,10) and C (32,36) on the plot grid. We would program as follows:

```
10 FOR X=10 TO 32 STEP 2
20 PLOT X,10
30 PLOT X,(13*X-20)/11
40 NEXT X
50 FOR X=32 TO 50 STEP 2
60 PLOT X,10
70 PLOT X,(740-13*X)/9
80 NEXT X
```

Now do this. When none of the lines joining the vertices are horizontal or vertical this will present little further difficulty. Take, for example, the triangle with vertices A (10,10), B (60,20) and C (34,38). The lines passing through AC, CB and AB are respectively:

$y=(7x-10)/16$, $y=(x+40)/5$, $y=(800-9x)/13$

(See the point slope formula in the Mathematical Appendix if you require more detail here.)

We program as follows:

```
10 FOR X=10 TO 34 STEP 2
20 PLOT X,(7*X-10)/6
30 PLOT X,(X+40)/5
40 NEXT X
50 FOR X=34 TO 60 STEP 2
60 PLOT X,(800-9*X)/13
70 PLOT X,(X+40)/5
80 NEXT X
```

The next simplest polygon is the quadrilateral. Let us draw one whose vertices are A (1,1), B (61,11), C (51,41) and D (33,33).

We now find the equations of the various line segments:

$y=(5+x)/6$ for AB, $y=194-3x$ for BC,
$y=(165+4x)/9$ for CD and $y=x$ for DA,

so we program as follows:

```
 10 FOR X=1 TO 61 STEP 2
 20 PLOT X,(5+X)/6
 30 NEXT X
 40 FOR X=61 TO 51 STEP -2
 50 PLOT X,194-3*X
 60 NEXT X
 70 FOR X=51 TO 33 STEP -3
 80 PLOT X,(165+4*X)/9
 90 NEXT X
100 FOR X=33 TO 1 STEP -3
110 PLOT X,X
120 NEXT X
```

For polygons of more than four vertices we use a similar approach. However, the regular polygons, i.e. those which have equal sides, require some further attention. To draw any regular polygon first place the centre of the polygon in the centre of the screen, which is (32,22) of the plot grid.

For the pentagon we should then have the following five vertices:

A_1 $(32+a,\ 22)$, A_2 $\left(32 + a\cos\frac{2\pi}{5},\ 22 + a\sin\frac{2\pi}{5}\right)$,

A_3 $\left(32 + a\cos\frac{4\pi}{5},\ 22 + a\sin\frac{4\pi}{5}\right)$,

A_4 $\left(32 + a\cos\frac{6\pi}{5},\ 22 + a\sin\frac{6\pi}{5}\right)$,

A_5 $\left(32 + a\cos\frac{8\pi}{5},\ 22 + a\sin\frac{8\pi}{5}\right)$,

with a being the radius of the circle which circumscribes the pentagon.

We note that the x coordinates of A_2 and A_5 coincide as well as those of A_3 and A_4, and we may use symmetry in our programming as the

pentagon has a line of symmetry parallel to the x axis passing through A_1. A useful radius is $a=22$. So we may program as follows:

```
 10 FOR X=54 TO 39 STEP -2
 20 PLOT X,96.322-1.3763*X
 30 PLOT X,1.3763*X-52.322
 40 NEXT X
 50 FOR X=39 TO 14 STEP -2
 60 PLOT X,0.3249*X+30.318
 70 PLOT X,13.684-0.3249*X
 80 NEXT X
 90 FOR Y=35 TO 9 STEP -2
100 PLOT 14,Y
110 NEXT Y
```

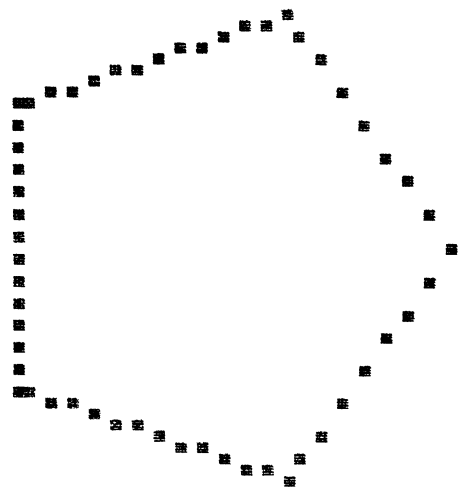


Figure 7.1

(See Figure 7.1.) For the regular hexagon the vertices are very simple:

$$A_1(32 + a,22),\ A_2(32 + \frac{a}{2},\ 22 + \frac{a\sqrt{3}}{2}),$$

$$A_3(32 - \frac{a}{2},\ 22 + \frac{a\sqrt{3}}{2}),$$

$A_4(32 - a, 22)$, $A_5\left(32 - \frac{a}{2}, 22 - \frac{a\sqrt{3}}{2}\right)$,

$A_6\left(32 + \frac{a}{2}, 22 - \frac{a\sqrt{3}}{2}\right)$.

Taking $a=20$, say, we have little trouble in drawing this:

```
 10 FOR X=52 TO 42 STEP -2
 20 PLOT X,112.067-1.732*X
 30 PLOT X,1.732*X-68.067
 40 NEXT X
 50 FOR X=42 TO 22 STEP -2
 60 PLOT X,39
 70 PLOT X,5
 80 NEXT X
 90 FOR X=22 TO 12 STEP -2
100 PLOT X,1.732*X+1.215
110 PLOT X,42.785-1.732*X
120 NEXT X
```

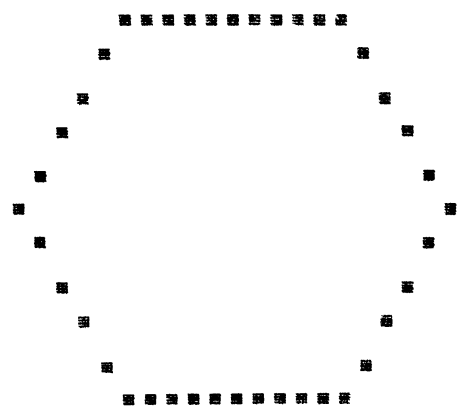


Figure 7.2

(See Figure 7.2.) Try this yourself.

Problems

1. Write a program to graph the triangle whose vertices are A (5,40), B (5,10) and C (50,20) on the plot grid.

2. Write a program to graph a flagpole with a triangular flag whose corner points are the three points A,B,C of problem 1. Darken the interior of the flag. (See Figure 7.3.)

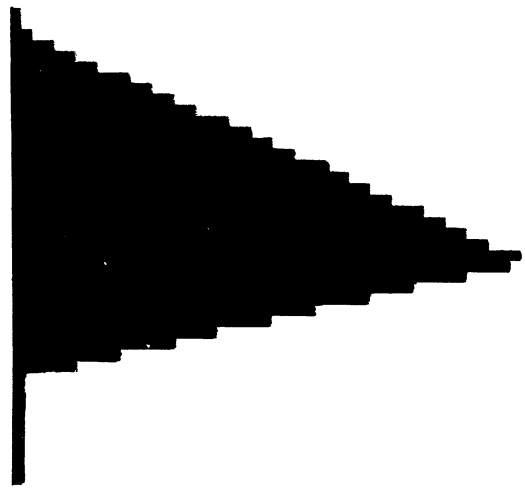


Figure 7.3

3. Write a program to graph the triangle whose vertices are A (5,40), B (10,10) and C (55,25).

4. Fill in triangle ABC of problem 3 with a checkerboard design. (See Figure 7.4.)

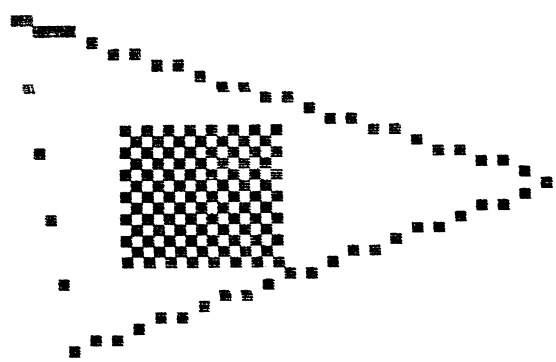


Figure 7.4

5. Write a program to draw a flagpole with a moving flag. (See Figure 7.5.)

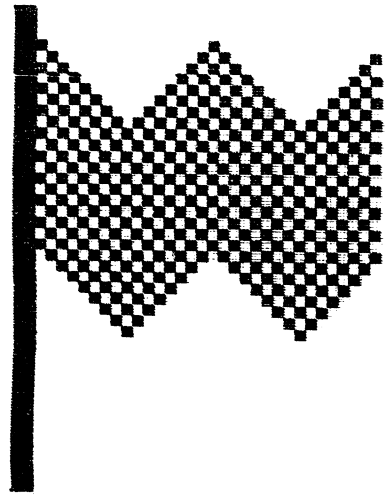


Figure 7.5

6. Fill in an equilateral triangle with a checkerboard design. (See Figure 7.6.)

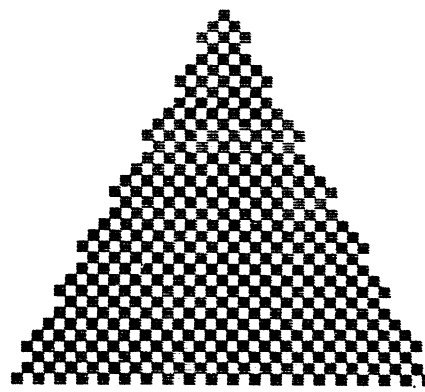


Figure 7.6

7. Write a program to draw two concentric regular pentagons, as shown in Figure 7.7.

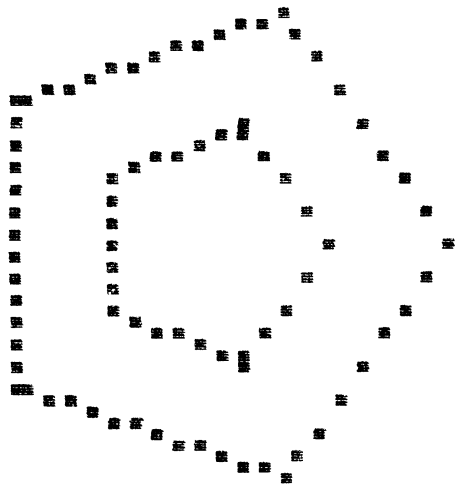


Figure 7.7

8. Write a program to draw a rocket as shown in Figure 7.8.

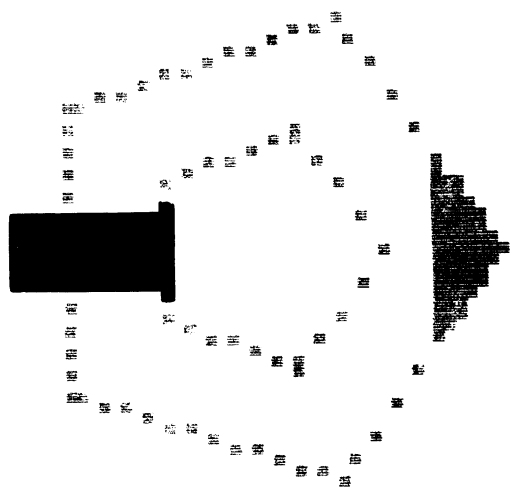


Figure 7.8

9. Fill in the regular pentagon shown in Figure 7.1 (radius=22, centre of figure at (32,22) and one point A_1 (54,22)). (See Figure 7.9.)

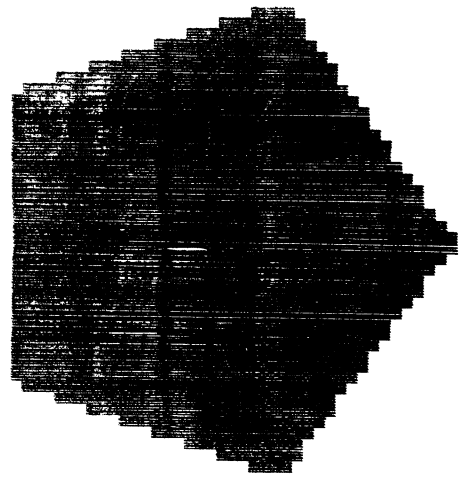


Figure 7.9

10. Fill in the regular hexagon shown in Figure 7.2 (radius =20, centre of figure at (32,22) and one point A_1 (52,22). (See Figure 7.10.)

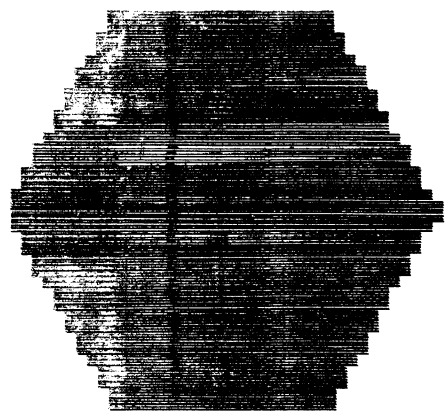


Figure 7.10

11. Design a program for a landmine. (See Figure 7.11.)
(Hint: Use the equation of a circle centred at (32,22) of radius 20.)

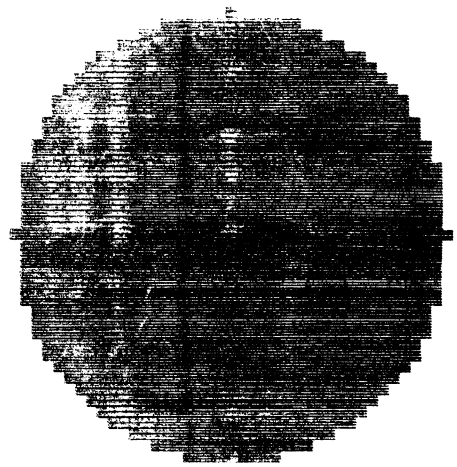


Figure 7.11

12. Write a program to draw a hexagonal tunnel as shown in Figure 7.12.

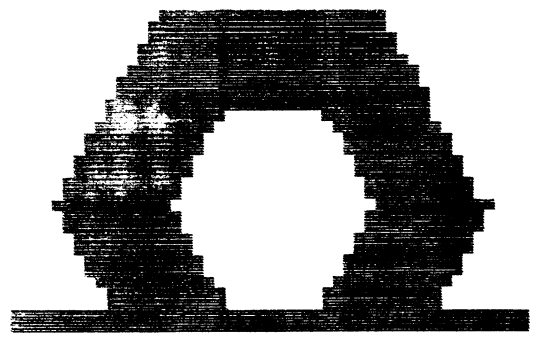


Figure 7.12

13. Write a program to draw a lifebelt as shown in Figure 7.13.

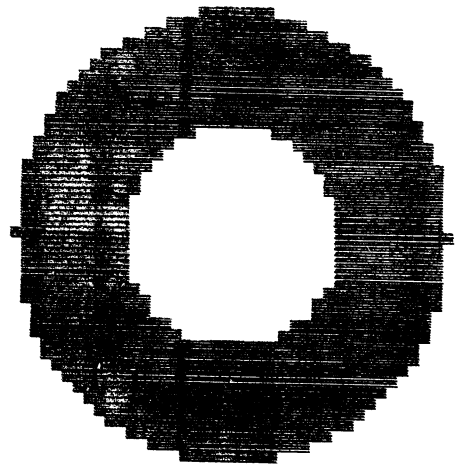


Figure 7.13

14. Fill in a regular octagon. (See Figure 7.14.)

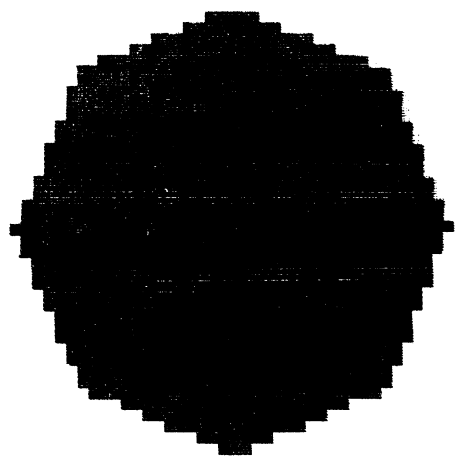


Figure 7.14

15. Design a program for a square mirror with an octagonal frame. (See Figure 7.15.)

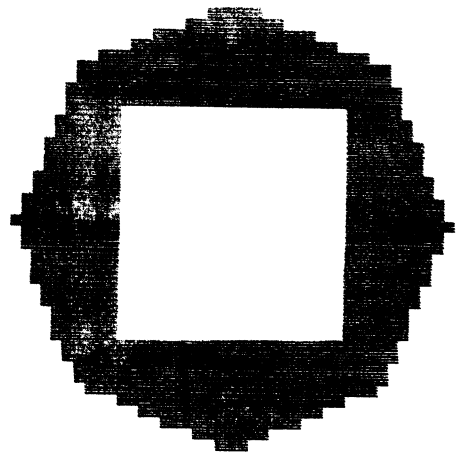


Figure 7.15

16. Write a program to draw a star of David. (See Figure 7.16.)

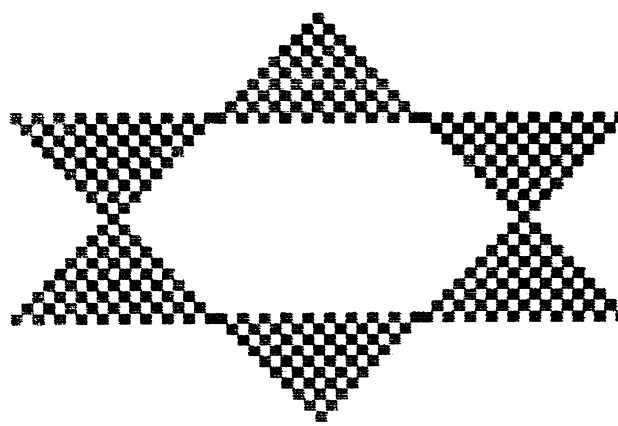


Figure 7.16

8 Drawing Pictures

When drawing pictures we generally use all the resources the T/S 1000 or ZX81 can offer — the plot grid, the letter grid and all the graphics symbols. What we can do is still very limited without the 16K RAM pack, and it is needed for drawing most of the pictures in this chapter.

The first step in designing any picture is to draw it on a suitable frame in pencil — this is a very important step. As we shall usually make use of both grids, the frame suggested in Figure 8.1 is appropriate. When you are satisfied with the picture you have drawn, ink it in. Now you must draw your design, section by section, on the screen. The following illustrations should make the procedure clear.

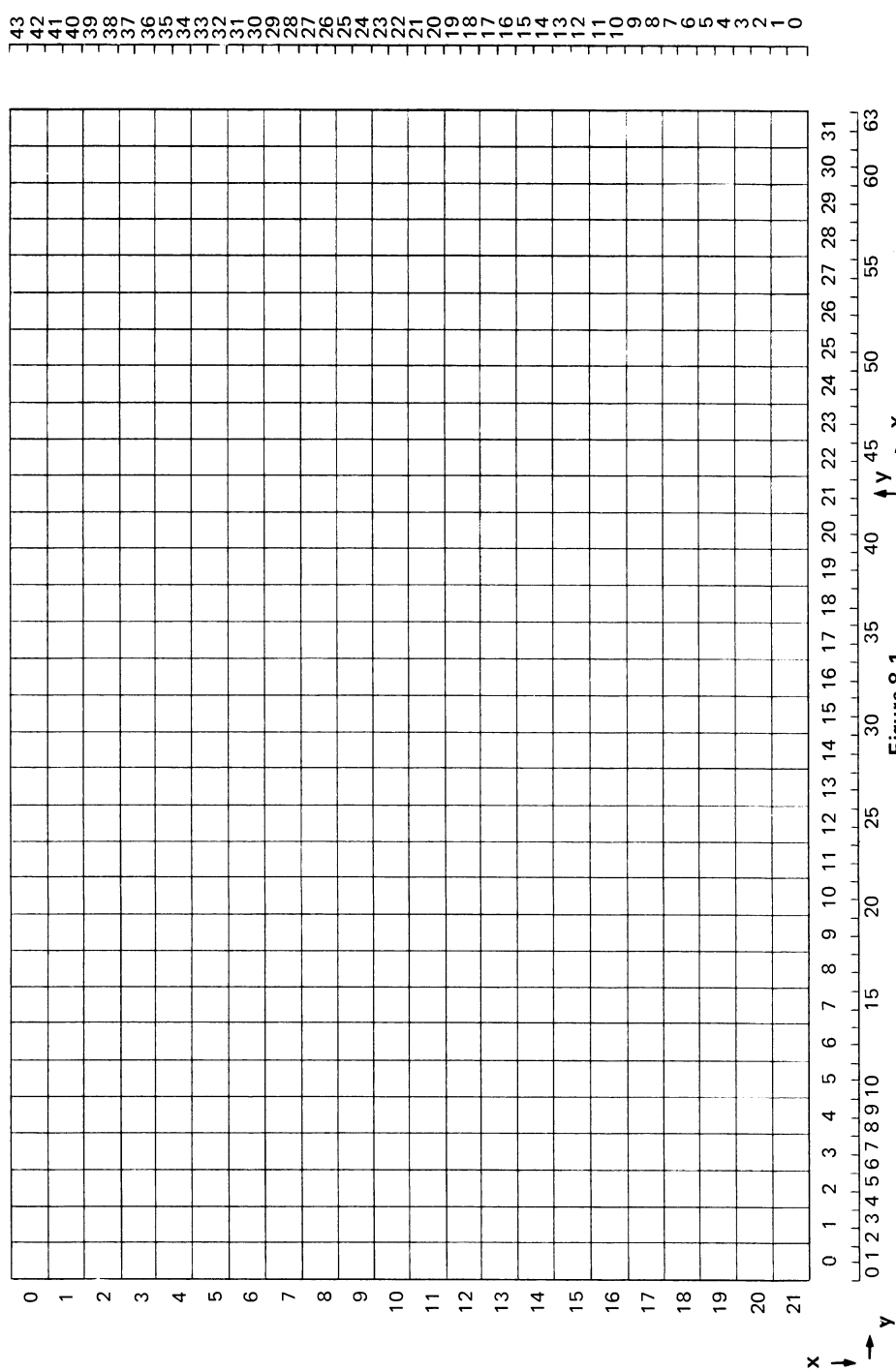
1. Drawing a table

After some trial and error in pencil we could come up with a design like the one in Figure 8.2. Now we will work out how to draw it on the screen in stages.

Stage 1: The table top.
Stage 2: The two front legs.
Stage 3: The left back leg.
Stage 4: The right back leg.

For the table top, it would be best to use the plot grid. Enter this program:

```
10 LET K=-1
20 FOR Y=14 TO 24
30 LET K=K+1
40 FOR X=18+K TO 40+K
50 PLOT X,Y
60 NEXT X
70 NEXT Y
```


0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43

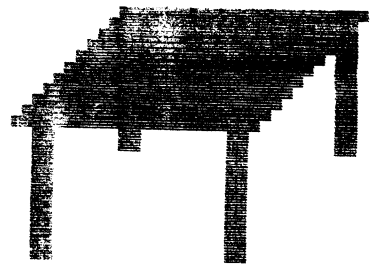


Figure 8.2

The remaining three stages deal with the four legs, so we shall write a little subroutine. The drawing of the legs suggests that the letter grid would be better here, so we write:

```
400 FOR Z=0 TO 3
410 PRINT AT A-Z,B;CHR$ 128
420 NEXT Z
430 RETURN
```

We are now ready to tackle the two front legs:

```
 75 FOR N=0 TO 9 STEP 9
 80 LET B=10+N
 90 LET A=20
100 GOSUB 400
110 PRINT AT 16,10+N;CHR$ 128
120 PRINT AT 15,10+N;CHR$ 128
130 NEXT N
```

Note that the N loop takes care of both front legs. Stage 3, the left back leg, is simple. We merely add

```
140 PRINT AT 15,14;CHR$ 128
```

For the last stage, the right back leg, we use the letter grid subroutine together with a suitable plot grid command:

```
150 LET A=15
160 LET B=24
170 GOSUB 400
180 PRINT AT 11,24;CHR$ 128
190 PLOT 49,22
200 GOTO 431
```

That's it.

2. Drawing a chair

After some trial and error with a pencil we might come up with a design like the one in Figure 8.3. There are another six stages necessary to construct this chair.

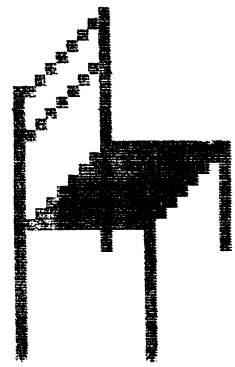


Figure 8.3

Stage 1: Write a program for the seat of the chair.

```
10 LET K=-1
20 FOR Y=22 TO 29
30 LET K=K+1
40 FOR X=21+K TO 32+K
50 PLOT X,Y
60 NEXT X
70 NEXT Y
```

Stage 2: The two front legs.

```
 80 FOR N=0 TO 12 STEP 12
 90 FOR Y=10 TO 21
100 PLOT 20+N,Y
110 NEXT Y
120 NEXT N
```

Stage 3: The left back leg.

```
130 PRINT AT 11,14;CHR$ 5
```

Stage 4: The right back leg.

```
140 FOR Y=20 TO 29
150 PLOT 39,Y
160 NEXT Y
```

Stage 5: The two back supports.

```
170 FOR N=0 TO 8 STEP 8
180 IF N=0 THEN LET B=22
190 IF N=8 THEN LET B=30
200 FOR Y=B TO B+12
210 PLOT 20+N,Y
220 NEXT Y
230 NEXT N
240 UNPLOT 28,42
```

Stage 6: The two cross bars.

```
250 FOR K=0 TO 6
260 PLOT 21+K,30+K
270 PLOT 21+K,34+K
280 NEXT K
```

(Replacing line 40 by 40 FOR X=21+K TO 32+K STEP 2 makes a nice cane chair. Try it.)

3. Drawing a steamer

Let's try to write a program to make a steamer like the one in Figure 8.4.

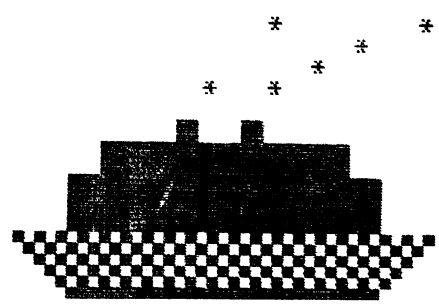


Figure 8.4

There are a lot of horizontal lines in this design, so a subroutine for drawing horizontal lines is needed.

Stage 1: Subroutine.

```
500 FOR Z=A TO B STEP C
510 PLOT Z,Y
520 NEXT Z
530 RETURN
```

Stage 2: Drawing the ship's hull.

```
 10 LET A=16
 20 LET B=44
 30 LET C=1
 40 LET Y=10
 50 GOSUB 500
 60 LET K=0
 65 LET M=0
 70 FOR Y=15-K TO 11-K STEP -1
 80 LET M=M+1
 90 LET A=10+M
100 LET B=50-M
110 LET C=2
120 GOSUB 500
130 NEXT Y
```

Stage 3: The first deck of the boat.

```
140 FOR Y=20 TO 16 STEP -1
150 LET A=16
160 LET B=44
170 LET C=1
180 GOSUB 500
190 NEXT Y
```

Stage 4: The second deck of the boat.

```
200 FOR Y=23 TO 21 STEP -1
210 LET A=19
220 LET B=41
230 GOSUB 500
240 NEXT Y
```

Stage 5: Both chimney stacks.

```
250 PRINT AT 9,13;CHR$ 128
260 PRINT AT 9,16;CHR$ 128
```

Stage 6: The smoke clouds.

```
270 PRINT AT 7,14;CHR$ 23
280 PRINT AT 4,17;CHR$ 23
290 PRINT AT 7,17;CHR$ 23
300 PRINT AT 6,19;CHR$ 23
310 PRINT AT 5,21;CHR$ 23
320 PRINT AT 4,24;CHR$ 23
330 GOTO 531
```

4. A house

Let's build a small house like the one in Figure 8.5. The picture contains quite a lot of vertical and horizontal lines so it will pay us to write a subroutine for both.

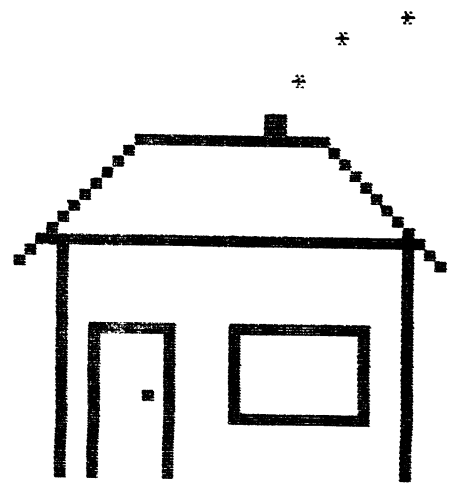


Figure 8.5

Stage 1: Subroutine.

```
500 FOR Z=A TO B
510 PLOT Z,Y
520 NEXT Z
530 RETURN
600 FOR Z=A TO B
610 PLOT X,Z
620 NEXT Z
630 RETURN
```

Stage 2: We shall begin by drawing all the horizontal lines.

```
 10 LET A=24
 20 LET B=41
 30 LET Y=31
 40 GOSUB 500
 50 LET A=16
 60 LET B=49
 70 LET Y=22
 80 GOSUB 500
 90 LET A=20
100 LET B=27
110 LET Y=14
120 GOSUB 500
130 LET A=33
140 LET B=45
150 GOSUB 500
160 LET Y=6
170 GOSUB 500
```

Stage 3: Now we shall draw all the vertical straight lines.

```
180 LET A=1
190 LET B=21
200 LET X=17
210 GOSUB 600
220 LET B=13
230 LET X=20
240 GOSUB 600
250 LET X=27
260 GOSUB 600
270 LET A=7
280 LET X=33
290 GOSUB 600
300 LET X=45
310 GOSUB 600
312 LET A=1
314 LET B=21
316 LET X=49
318 GOSUB 600
```

Stage 4: The door knob on the house, and the chimney stack.

```
320 PLOT 25,8
330 PRINT AT 5,18;CHR$ 128
```

Stage 5: The left slope of the roof.

```
340 LET K=-1
350 FOR X=13 TO 23
360 LET K=K+1
370 PLOT 13+K,20+K
380 NEXT X
```

Stage 6: The right slope of the roof.

```
390 LET K=-1
400 FOR X=42 TO 52
410 LET K=K+1
420 PLOT 42+K,30-K
430 NEXT X
```

Stage 7: The smoke from the chimney.

```
440 PRINT AT 3,19;CHR$ 23
450 PRINT AT 1,21;CHR$ 23
460 PRINT AT 0,24;CHR$ 23
470 GOTO 631
```

5. A weaving pattern (Figure 8.6)

In this drawing there is a single pattern repeated 35 times, so clearly a subroutine is needed. A suitable one is as follows.

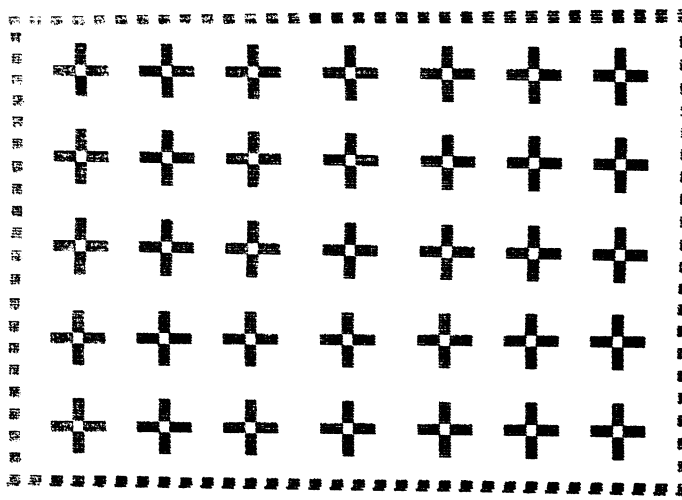


Figure 8.6

Stage 1: Subroutine.

```
500 FOR Z=0 TO 4
510 PLOT X,Y+Z
520 NEXT Z
530 FOR Z=0 TO 4
```

```
640 PLOT X+Z-2,Y+2
650 NEXT Z
660 UNPLOT X,Y+2
670 RETURN
```

Stage 2: The fringe of the design.

```
 10 FOR Y=0 TO 42 STEP 42
 20 FOR X=0 TO 62 STEP 2
 30 PLOT X,Y
 40 NEXT X
 50 NEXT Y
 60 FOR X=0 TO 62 STEP 62
 70 FOR Y=0 TO 40 STEP 2
 80 PLOT X,Y
 90 NEXT Y
100 NEXT X
```

Stage 3: Plot the set of 15 crosses on the left-hand side of the screen and the 15 crosses on the right-hand side.

```
105 FOR K=0 TO 34 STEP 34
110 FOR Y=3 TO 35 STEP 8
120 FOR X=K+6 TO K+22 STEP 8
130 GOSUB 600
140 NEXT X
150 NEXT Y
155 NEXT K
```

Stage 4: Do the central column of crosses.

```
160 LET X=31
170 FOR Y=3 TO 35 STEP 8
180 GOSUB 600
190 NEXT Y
200 GOTO 680
```

What can be done on the plot grid can be even more effectively done on the letter grid.

6. Another weaving pattern (Figure 8.7)

In this drawing there are 12 identical patterns, so a subroutine is needed again.

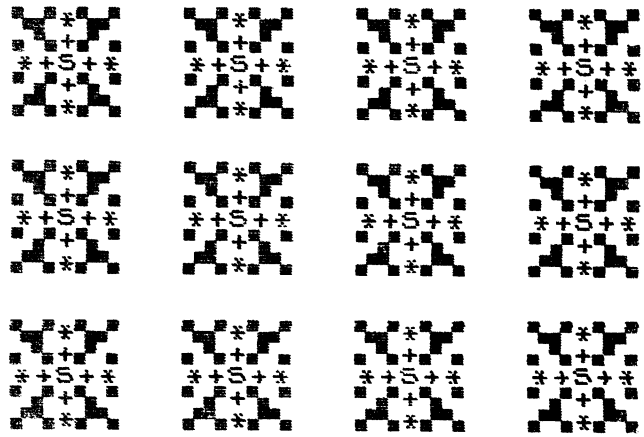


Figure 8.7

Stage 1: Subroutine.

```
10 LET A$=CHR$ 134
20 LET B$=CHR$ 6
30 LET C$=CHR$ 23
40 LET D$=CHR$ 4
50 LET E$=CHR$ 21
60 LET F$=CHR$ 135
70 LET G$=CHR$ 56
80 LET H$=CHR$ 1
90 LET I$=CHR$ 2
 600 PRINT AT X,Y;A$;B$;C$;A$;B$
 610 PRINT AT X+1,Y;D$;A$;E$;B$;
F$
 620 PRINT AT X+2,Y;C$;E$;G$;E$;
C$
 630 PRINT AT X+3,Y;H$;B$;E$;A$;
I$
 640 PRINT AT X+4,Y;B$;A$;C$;B$;
A$
 650 RETURN
```

Stage 2: Placement for all 12 patterns.

```
100 FOR Y=2 TO 26 STEP 8
110 FOR X=2 TO 16 STEP 7
120 GOSUB 600
130 NEXT X
140 NEXT Y
150 GOTO 660
```

And that's all this time.

Problems

1. Write a program to make a cupboard as shown in Figure 8.8.

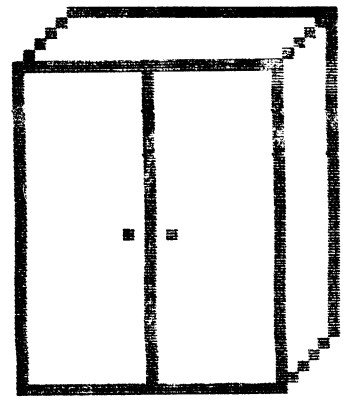


Figure 8.8

2. Write a program to draw a chest of drawers as shown in Figure 8.9.

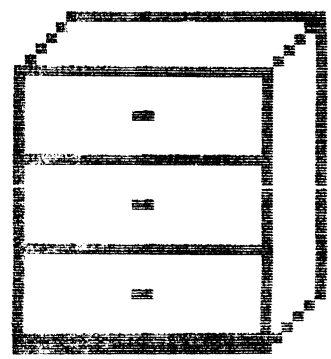


Figure 8.9

3. Write a program to make a banner like the one shown in Figure 8.10.

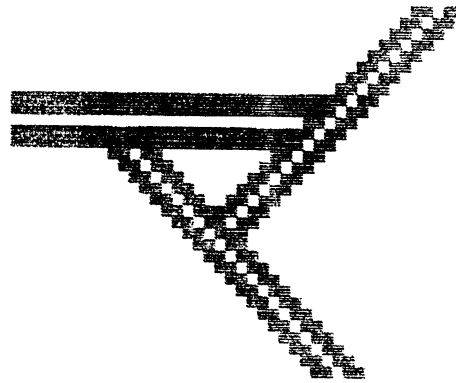


Figure 8.10

4. Draw a simplified version of the American flag. (See Figure 8.11.)

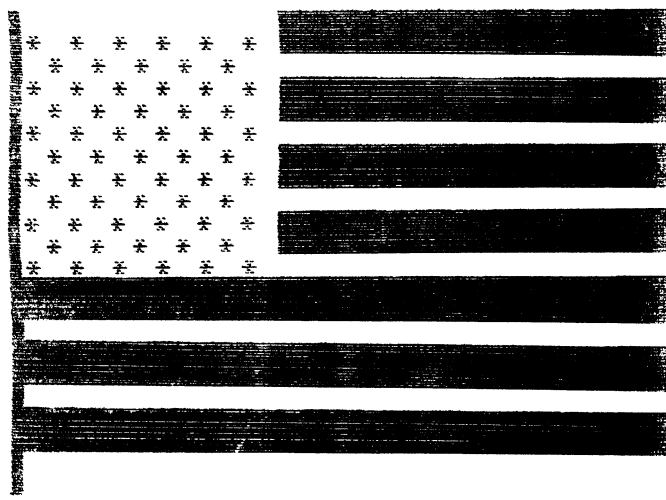


Figure 8.11

5. Write a program to draw a dressing table as shown in Figure 8.12.

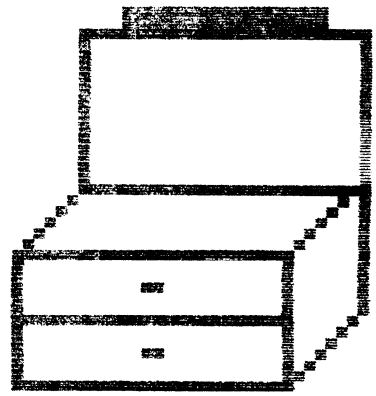


Figure 8.12

6. Draw a book as shown in Figure 8.13.

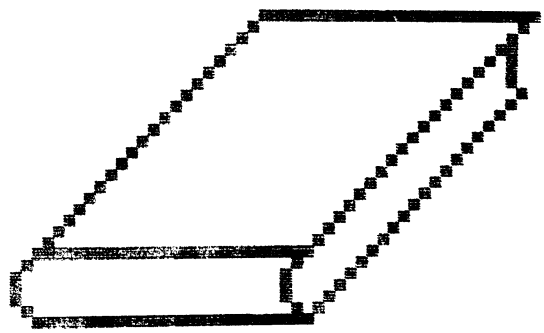


Figure 8.13

7. Write a program to draw a clown's hat with a feather in the top as given in Figure 8.14.

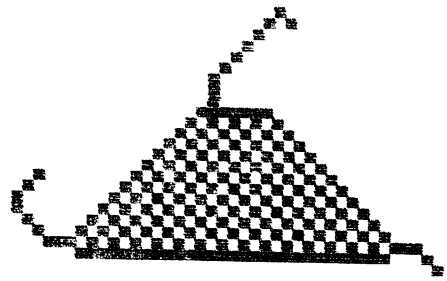


Figure 8.14

8. Do a simple crossword design as given in Figure 8.15.

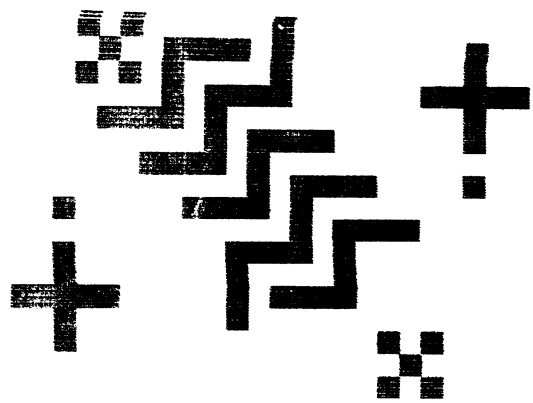


Figure 8.15

9. Write a program to design a radiator as shown in Figure 8.16.

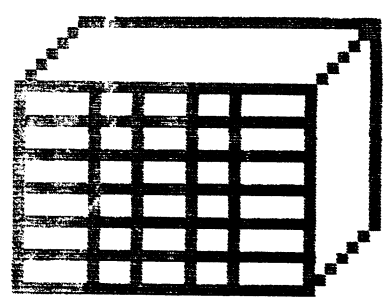


Figure 8.16

10. Write a program to draw a kitchen pot. (See Figure 8.17.)

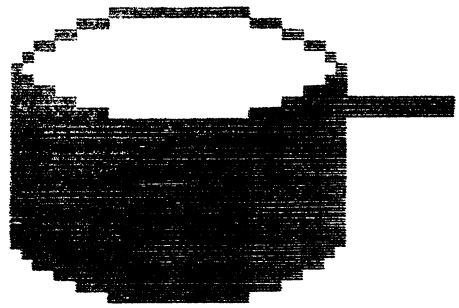


Figure 8.17

11. Write a program to draw a flower vase as shown in Figure 8.18.

Figure 8.18

12. Write a program to design a drinking mug as shown in Figure 8.19.

Figure 8.19

13. Write a program for a champagne glass as shown in Figure 8.20.

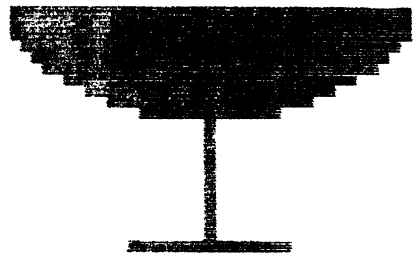


Figure 8.20

14. Write a program to draw a wine bottle as given in Figure 8.21.

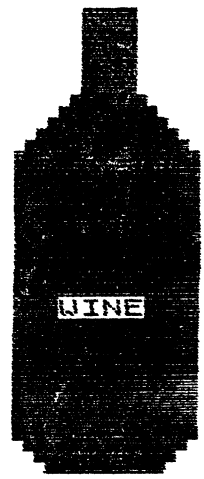


Figure 8.21

15. Design a washing machine as shown in Figure 8.22.

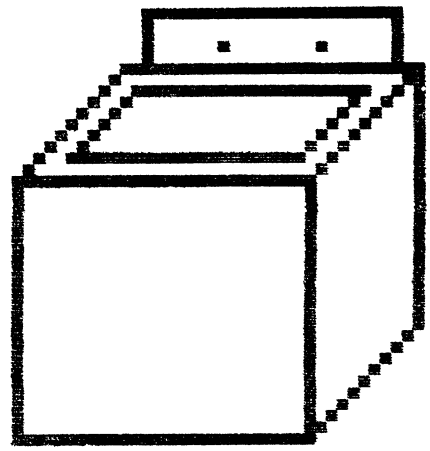


Figure 8.22

16. Draw a watering can as given in Figure 8.23.

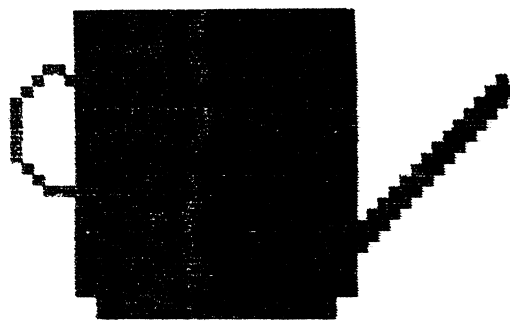


Figure 8.23

17. Write a program to design a T.V. set as shown in Figure 8.24.

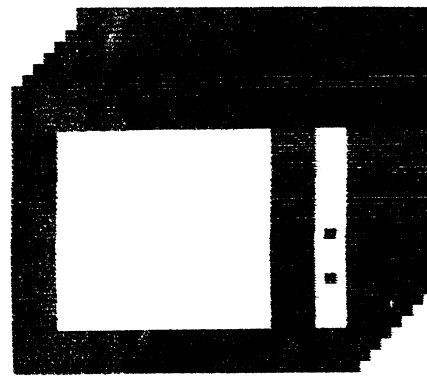


Figure 8.24

18. Write a program to draw the face shown in Figure 8.25.

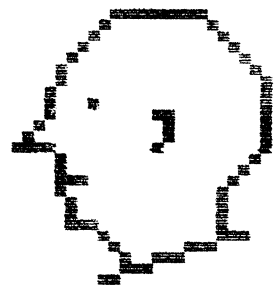


Figure 8.25

19. Draw the back of a man's head as given in Figure 8.26.

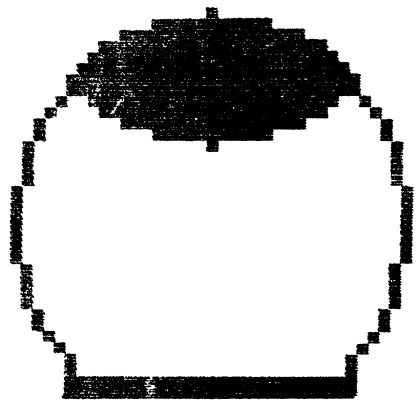


Figure 8.26

20. Draw a guitar as shown in Figure 8.27.

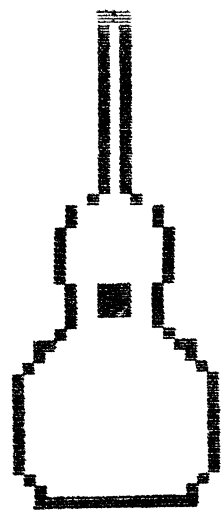


Figure 8.27

21. Design a ladder as given in Figure 8.28.

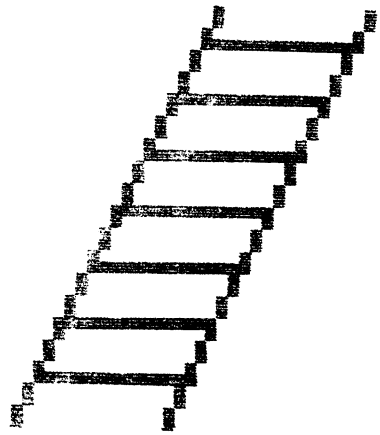


Figure 8.28

22. Draw a post-box as shown in Figure 8.29.

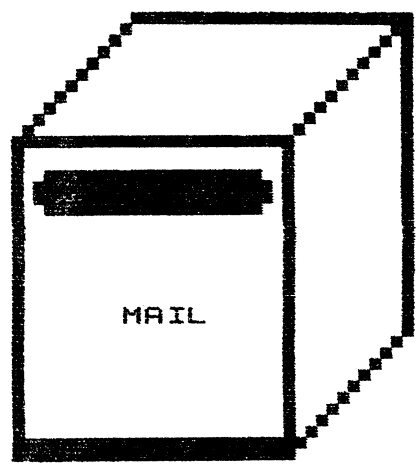


Figure 8.29

23. Design a program to produce the plane as shown in Figure 8.30.

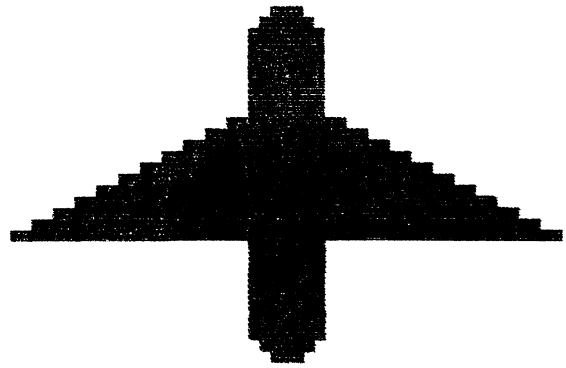


Figure 8.30

24. Make up a program to do the washing basket as drawn in Figure 8.31.

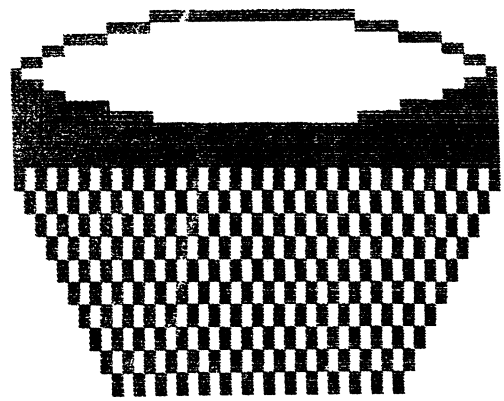


Figure 8.31

25. Design a cart as shown in Figure 8.32.

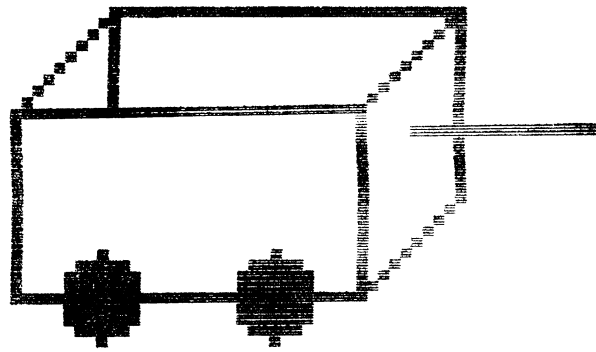


Figure 8.32

26. Write a program for the sailboat as shown in Figure 8.33.

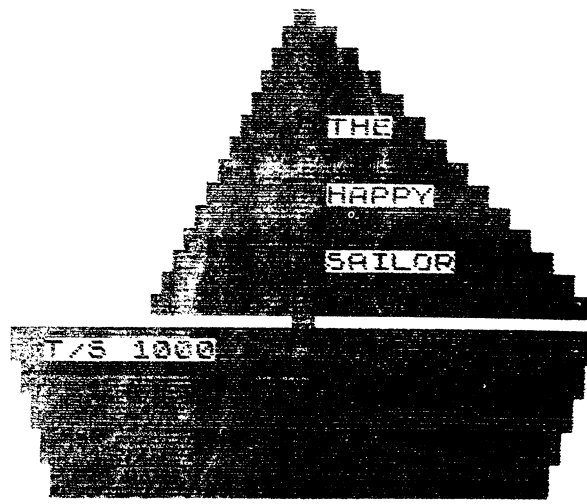


Figure 8.33

27. Design a program for a rocket as shown in Figure 8.34.

Figure 8.34

28. Write a program to draw a car as shown in Figure 8.35.

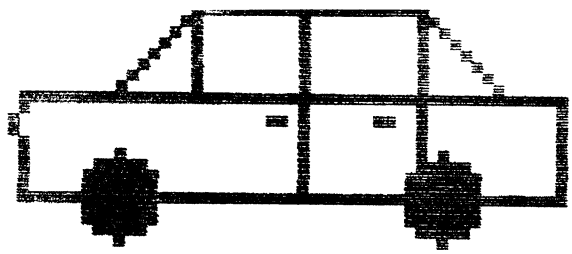


Figure 8.35

29. Write a program for the cat shown in Figure 8.36.

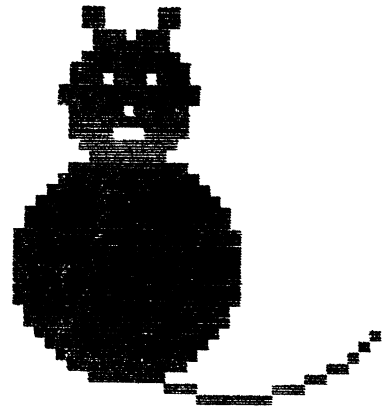


Figure 8.36

30. Write a program for a tunnel as given in Figure 8.37.

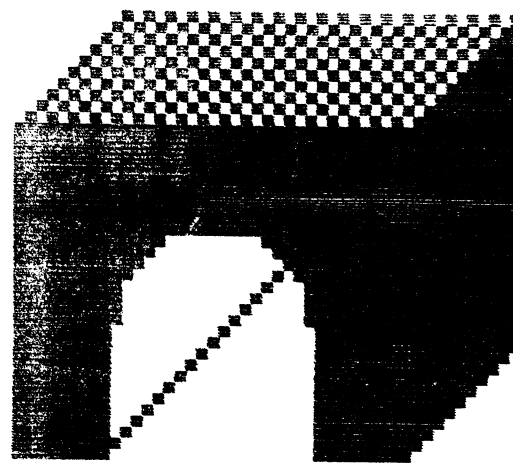


Figure 8.37

31–40. Make the weaving patterns shown in Figures 8.38–8.47.

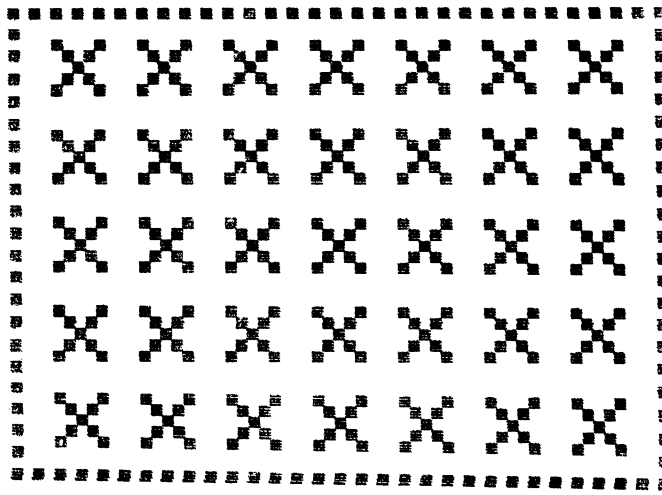


Figure 8.38

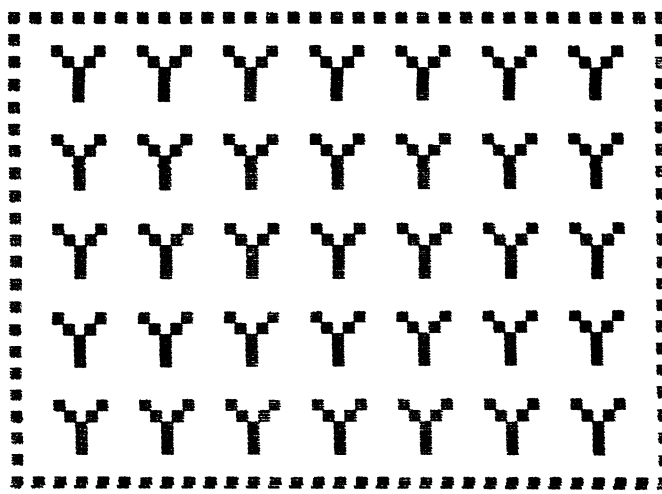


Figure 8.39

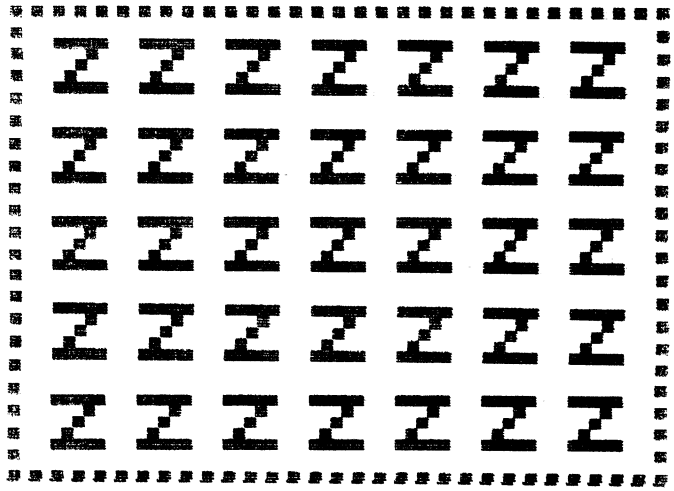


Figure 8.40

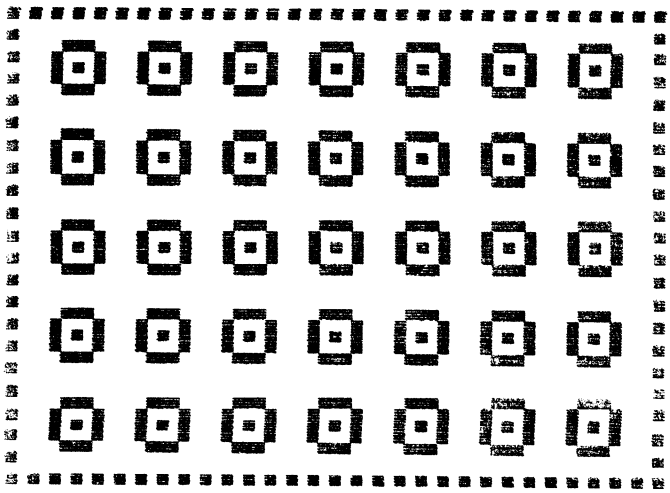


Figure 8.41

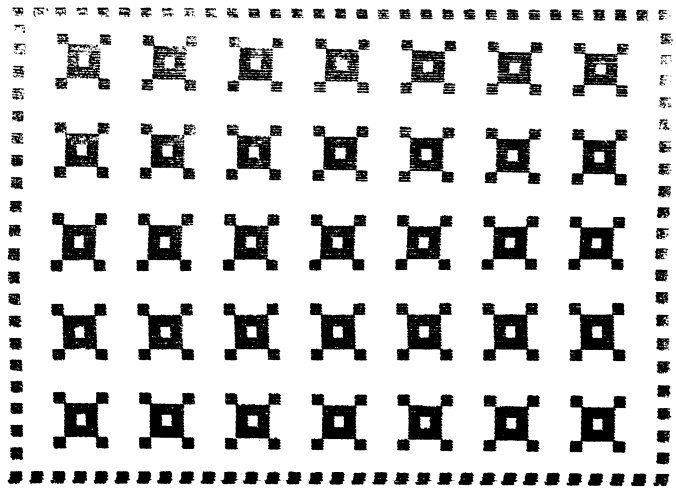


Figure 8.42

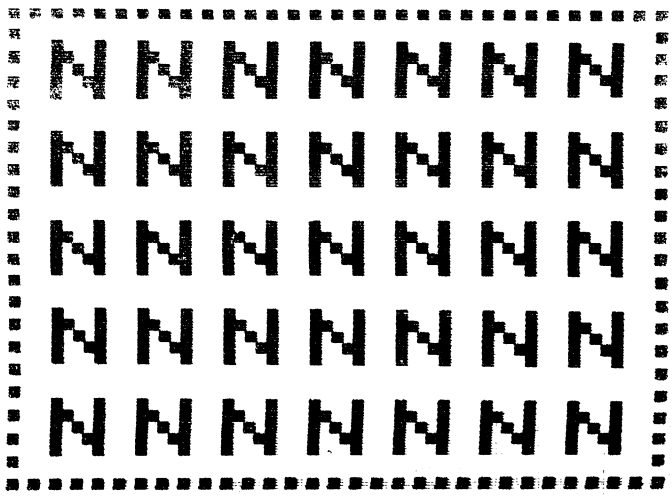


Figure 8.43

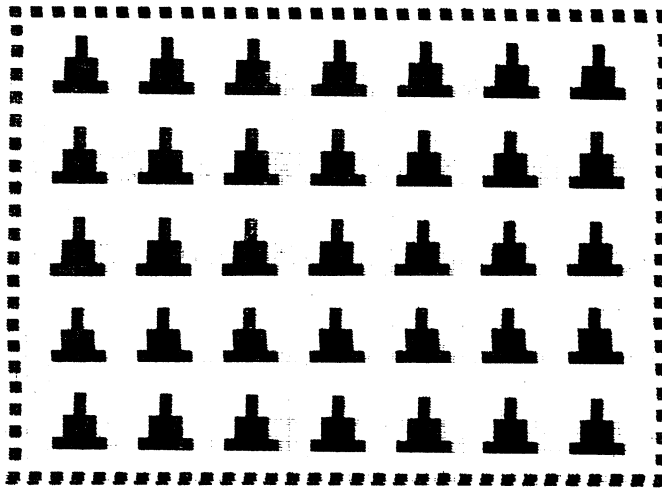


Figure 8.44

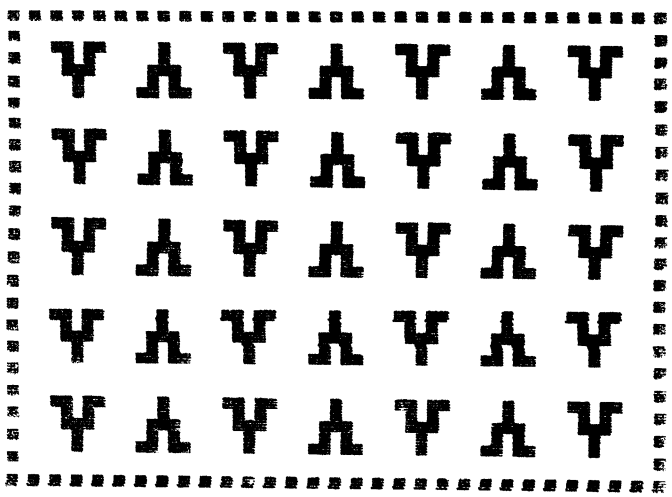


Figure 8.45

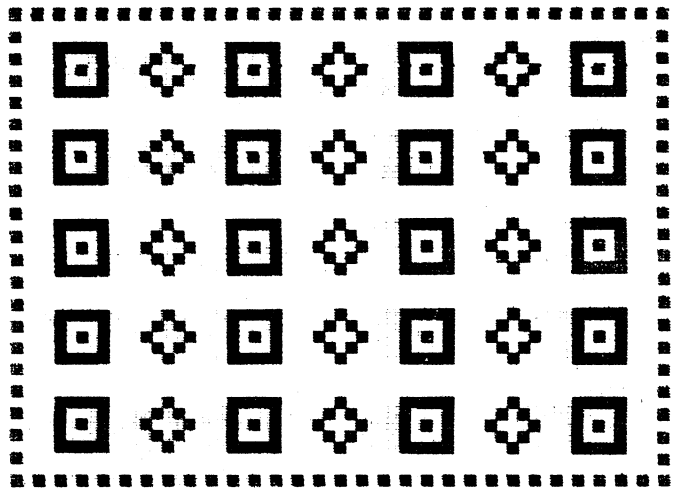


Figure 8.46

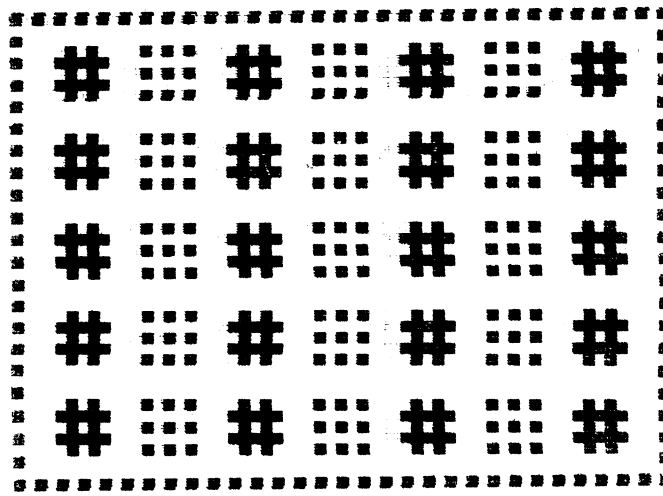


Figure 8.47

41–50. Use the letter grid only to produce the weaving patterns in Figures 8.48–8.57.

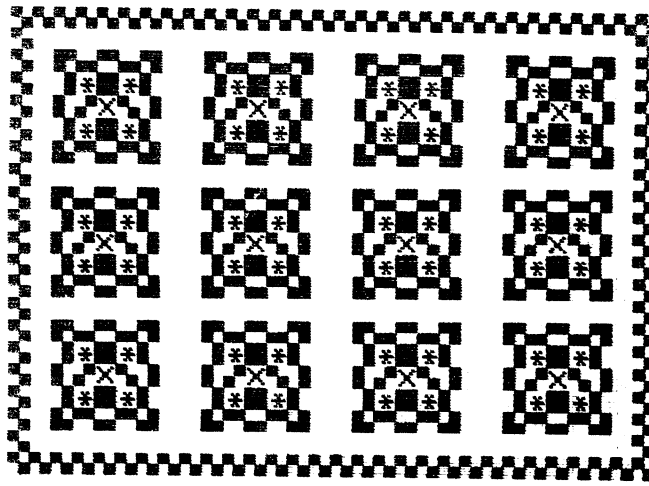


Figure 8.48

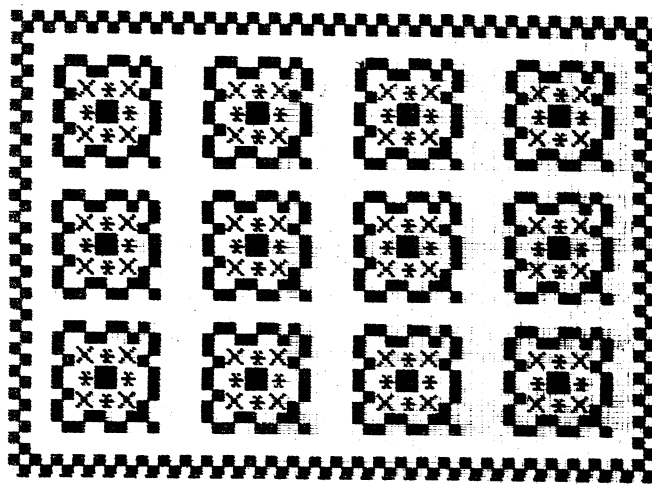


Figure 8.49

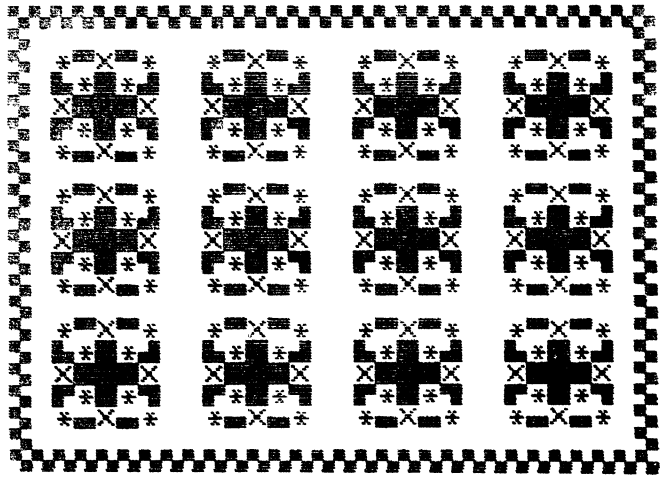


Figure 8.50

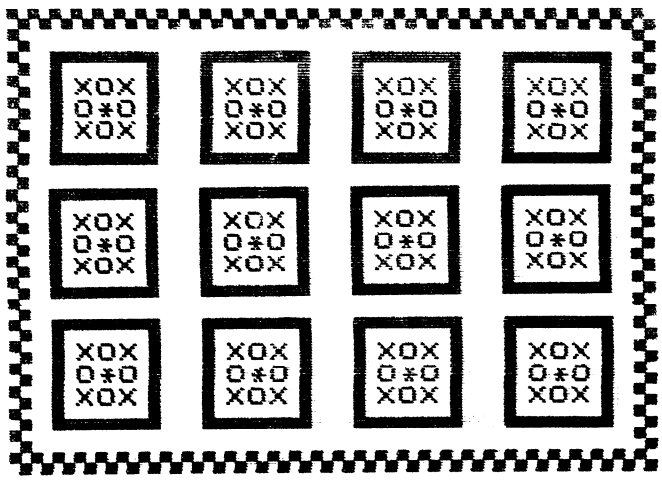


Figure 8.51

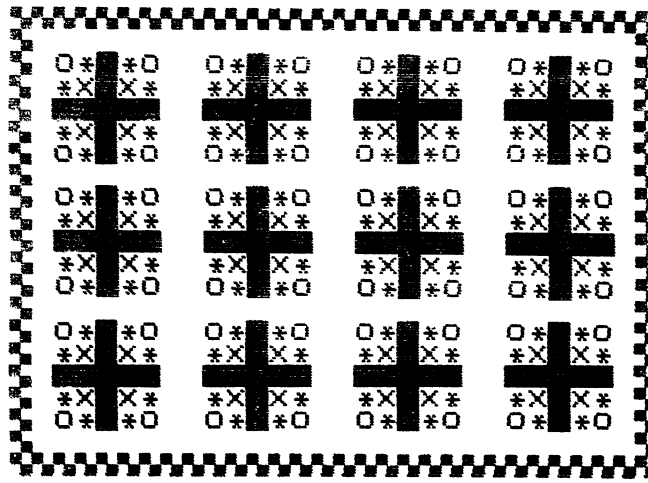


Figure 8.52

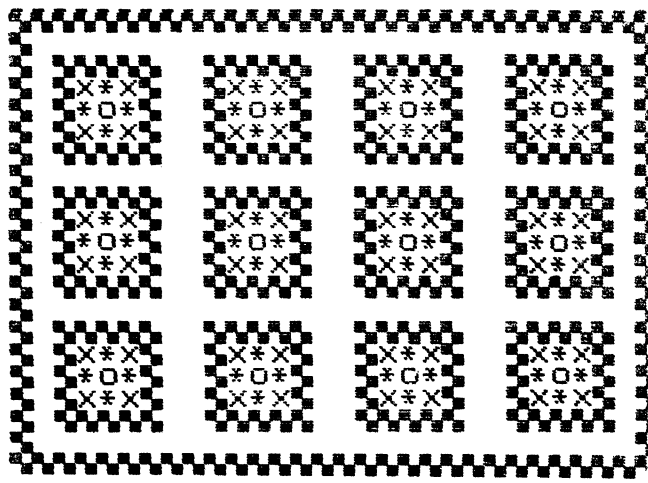


Figure 8.53

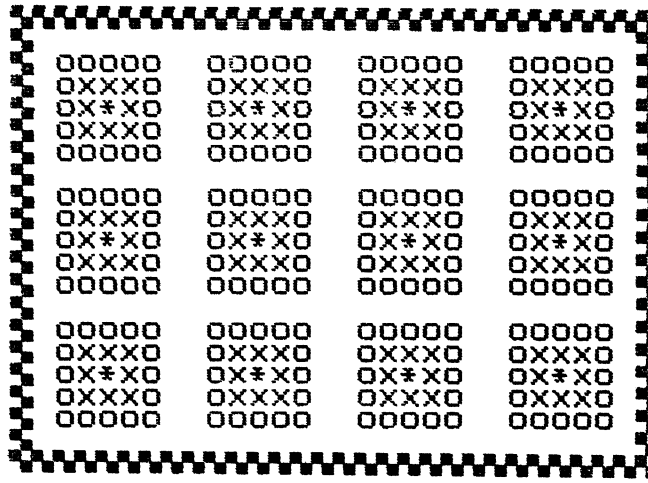


Figure 8.54

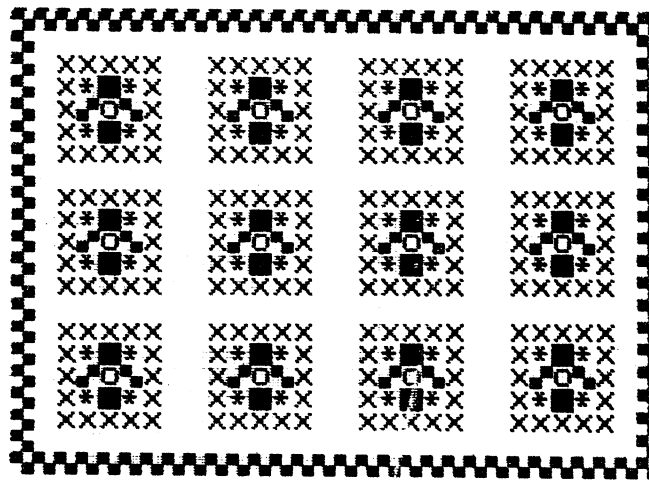


Figure 8.55

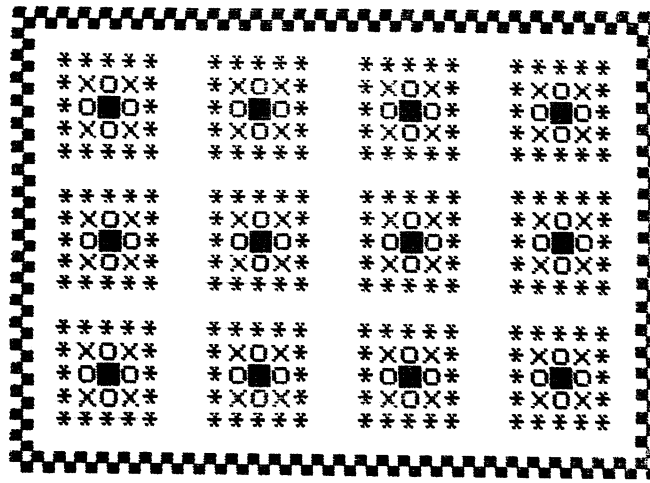


Figure 8.56

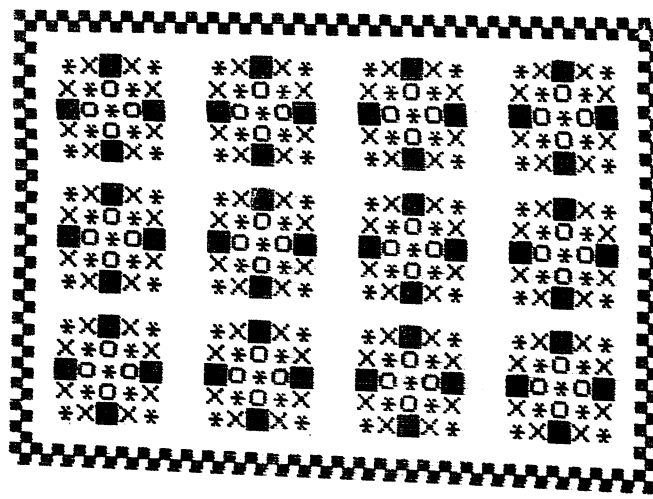


Figure 8.57

9 Motion

The computer can only imitate motion. The simplest way to achieve this effect is to use the PLOT and UNPLOT commands, i.e. if we wish to go from left to right we use first PLOT then UNPLOT in sequence, which gives the illusion of motion. Let us try this out using the "flashing pixel" in some of the drawings we have already designed. The pixel may flash on and off in the same position or may move across the screen.

1. The cat in Figure 8.36 (page 74)

(a) Suppose we want to make the cat in Figure 8.36 wink its left eye. Begin with the program which produces the cat, then program the wink as follows:

```
440 FOR K=1 TO 50
450 PLOT 29,33
460 GOSUB 600
470 UNPLOT 29,33
480 GOSUB 600
490 NEXT K
500 GOTO 630
600 FOR R=1 TO 50
610 NEXT R
620 RETURN
```

(b) Let's make the cat smile. Begin with the program steps to draw the cat, then add:

```
440 FOR K=1 TO 50
450 UNPLOT 25,29
460 UNPLOT 29,29
470 GOSUB 600
480 PLOT 25,29
490 PLOT 29,29
```

```
500 GOSUB 600
510 NEXT K
520 GOTO 530
600 FOR R=1 TO 35
610 NEXT R
620 RETURN
```

(c) Let's try to wiggle the cat's tail. This is a little harder to do as it involves quite a few pixel points. Repeat the cat program again. Then try the following to wiggle its tail:

```
440 FOR K=1 TO 30
450 FOR X=44 TO 50
460 LET D=INT 50R (74*X-X*X-110
0)
470 UNPLOT X,20-D
480 NEXT X
490 FOR X=44 TO 50
500 PLOT X,6
510 NEXT X
520 FOR X=44 TO 50
530 UNPLOT X,6
540 NEXT X
550 FOR X=44 TO 50
560 LET D=INT 50R (74*X-X*X-110
0)
570 PLOT X,20-D
580 NEXT X
590 NEXT K
```

2. The house in Figure 8.5 (page 56)

To make the smoke from the chimney move through the sky is fairly easy. First we must clear the smoke that is already there. This can be done as follows:

```
631 PRINT AT 3,19;CHR$ 0
640 PRINT AT 1,21;CHR$ 0
650 PRINT AT 0,24;CHR$ 0
```

Now we will put in some more smoke, this time a little to the right of where we put the last smoke:

```
660 FOR K=1 TO 30
670 FOR S=1 TO 7
680 PRINT AT 3,19+S;CHR$ 23
690 PRINT AT 1,21+S;CHR$ 23
700 PRINT AT 0,24+S;CHR$ 23
800 GOSUB 1000
810 PRINT AT 3,19+S;CHR$ 0
```

```
  820 PRINT AT 1,21+S;CHR$ 23
  830 PRINT AT 0,24+S;CHR$ 0
  840 GOSUB 1000
  850 NEXT S
  860 NEXT K
  870 GOTO 1030
 1000 FOR R=1 TO 30
 1010 NEXT R
 1020 RETURN
```

3. A slowly swinging pendulum bob

Assume that the pivot of the pendulum is at (32,22) and the bob describes part of the circle

$(x-32)^2 + (y-22)^2 = 400$
for which $18 \leqslant x \leqslant 46$.

(See the equation to a circle in the Mathematical Appendix.) Now program as follows:

```
   10 FOR I=1 TO 20
   20 IF I/2=INT (I/2) THEN GOTO
50
   30 FOR X=18 TO 46
   40 GOTO 60
   50 FOR X=46 TO 18 STEP -1
   60 LET D=INT SQR (64*X-X*X-624)
   70 PLOT X,22-D
   80 GOSUB 200
   90 UNPLOT X,22-D
  100 NEXT X
  110 NEXT I
  115 GOTO 230
  200 FOR K=1 TO 4
  210 NEXT K
  220 RETURN
```

You may wonder why we introduced the subroutine. Try to run the program without it and see what happens!

4. Traffic lights

For convenience of programming, let us make the three traffic lights squares rather than circles. We shall first show the top signal (go), then clear this and show the middle signal (caution), then clear this and show the bottom signal (stop).

This process may be programmed as follows:

```
  10 FOR K=1 TO 20
  20 FOR N=1 TO 6
  30 IF N/2=INT (N/2) THEN CLS
  40 IF N=1 THEN LET B=8
  50 IF N=3 THEN LET B=6
  60 IF N=5 THEN LET B=128
  70 IF N=1 OR N=2 THEN LET A=0
  80 IF N=3 OR N=4 THEN LET A=7
  90 IF N=5 OR N=6 THEN LET A=14
  95 IF N/2=INT (N/2) THEN GOTO
110
 100 GOSUB 300
 110 NEXT N
 130 NEXT K
 140 GOTO 370
 300 FOR X=A TO A+5
 310 FOR Y=13 TO 19
 320 PRINT AT X,Y;CHR$ B
 340 NEXT Y
 350 NEXT X
 352 IF N=1 THEN PRINT AT 3,15,"
STOP"
 354 IF N=3 THEN PRINT AT 10,13;
"CAUTION"
 356 IF N=5 THEN PRINT AT 17,15;
"GO"
 360 RETURN
```

5. A simple kaleidoscope

Again we will make the kaleidoscope a square rather than a circle, to make programming a little easier. First we will show one pattern in slow motion, then clear the kaleidoscope and show another pattern in slow motion. We will then repeat the pattern any number of times.

```
  10 LET A$=CHR$ 130
  20 LET B$=CHR$ 6
  30 FOR K=1 TO 20
  40 FOR I=0 TO 1
  50 FOR X=4 TO 17
  60 FOR Y=5 TO 18
  70 IF I=0 THEN GOTO 100
  80 PRINT AT X,Y;A$
  90 GOTO 110
 100 PRINT AT X,Y;B$
 110 NEXT Y
 120 NEXT X
 130 CLS
 140 NEXT I
 150 NEXT K
```

6. A more complex kaleidoscope

Using the square frame again, let us design a kaleidoscope with ten different simple patterns inside the frame. (See Figure 9.1.)

```
   20 FOR N=166 TO 164 STEP 2
   30 FOR X=3 TO 17 STEP 2
   40 LET M=N+1
   50 PRINT AT X,5;CHR$ N;CHR$ 23
;CHR$ M;CHR$ 23;CHR$ N;CHR$ 23;C
HR$ M;CHR$ 23;CHR$ N;CHR$ 23;CHR
$ M;CHR$ 23;CHR$ N;CHR$ 23;CHR$
M;CHR$ 23
   60 PRINT AT X+1,5;CHR$ 23;CHR$
 N;CHR$ 23;CHR$ M;CHR$ 23;CHR$ N
;CHR$ 23;CHR$ M;CHR$ 23;CHR$ N;C
HR$ 23;CHR$ M;CHR$ 23;CHR$ N;CHR
$ 23;CHR$ M
   70 NEXT X
   80 CLS
   90 NEXT N
```

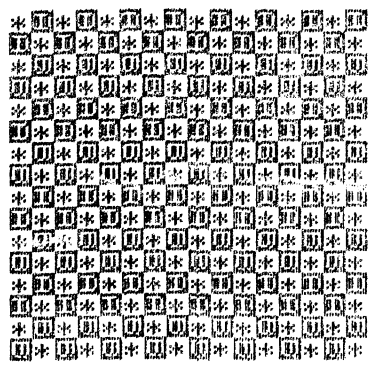


Figure 9.1

7. Shifting a petrol drum from the left of the screen to the far right

We are not interested in how the petrol drum is formed bit by bit, but want to begin with a whole drum, so we need to use FAST, PAUSE and POKE.

```
  5 FAST
 10 FOR X=0 TO 4
 20 FOR Y=0 TO 12
 30 PLOT X,Y
 40 NEXT Y
 50 NEXT X
 60 PAUSE 50
 70 POKE 16437,255
```

The above program gives the initial position of the drum on the left-hand side. We will now push it slowly, frame by frame, to the right:

```
  80 FOR K=0 TO 58
  90 LET A=1
 100 FOR X=K TO K+5 STEP 5
 110 LET A=A+1
 120 FOR Y=0 TO 12
 130 IF A/2=INT (A/2) THEN GOTO
160
 140 PLOT K+5,Y
 150 GOTO 170
 160 UNPLOT K,Y
 170 NEXT Y
 180 NEXT X
 190 PAUSE 50
 200 POKE 16437,255
 210 NEXT K
```

Problems

1. Go back to the dressing table in Figure 8.12 (page 63) and switch the mirror light on and off.

2. Go back to Figure 8.25 (page 68) and make the man wink his eye.

3. Make the man in Figure 8.25 wink his eye and grin at the same time.

4. Make the smoke from the liner in Figure 8.4 (page 54) move to the right.

5. Make the feather in the clown's hat in Figure 8.14 (page 64) flutter in the wind.

6. Make water come out of the spout of the can in Figure 8.23 (page 67).

7. Make the cart in figure 8.32 (page 72) appear to move from left to right.

8. Make the car in Figure 8.35 (page 73) appear to move from right to left.

9. Let the pixel frog jump around in a circle for twenty rounds.

10. Devise a program to simulate driving rain.

11. Make a simple kaleidoscope (as in illustration 5, page 88 of this chapter) with the following four different frames:

Frame 1 is a square consisting entirely of character code 23,
Frame 2 is a square consisting entirely of character code 134,
Frame 3 is a square consisting entirely of character code 13,
Frame 4 is a square consisting entirely of character code 21.

12. Make a simple kaleidoscope with two different frames as shown in Figure 9.2.

a. **b.**

Figure 9.2

13. Make a simple kaleidoscope with four different frames as shown in Figure 9.3.

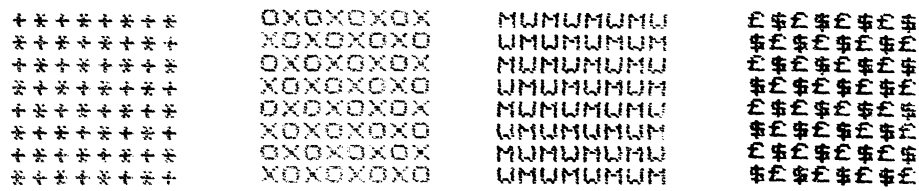


Figure 9.3

14. Make a simple kaleidoscope with four different frames as shown in Figure 9.4.

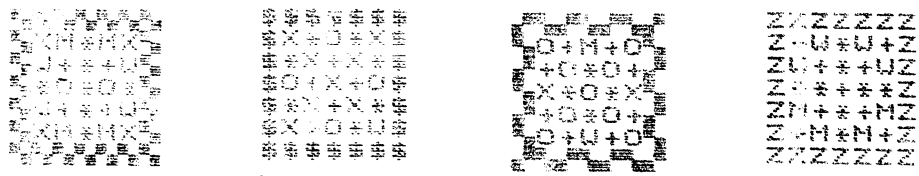


Figure 9.4

15. Modify problem 11 above to make a more complex kaleidoscope with the same four frames, but this time make the frames round.

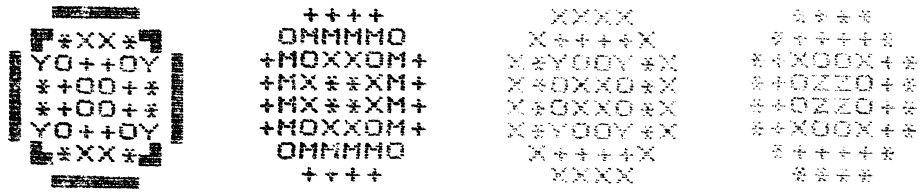
16. Do the same for problem 12.

17. Do the same for problem 13.

18. Do the same for problem 14.

19. Design a kaleidoscope as in problem 15 above with 26 different frames, each frame being filled entirely by repeating one letter of the English alphabet. (Hint: Use the same subroutine for each of the 26 frames.)

20. Design a kaleidoscope as in problem 15 with four different frames as shown in Figure 9.5.



a. **b.** **c.** **d.**

Figure 9.5

21. Slowly shift the table in Figure 9.6 from the left-hand side of the screen to the right. (Hint: Use a subroutine for the table.)

a. **b.**

Figure 9.6

10 A Collection of Interesting Curves

1. The Witch of Agnesi

This curve owes its name to Maria Gaetana Agnesi (1718–1799) who was a brilliant Italian mathematician and philosopher. Her best-known work was a popular calculus book written in Latin in which she discusses this curve. She called it *versiera* meaning the versed sine curve in Latin. Unfortunately in Italian *versiera* means she-devil, hence witch, so this curve got its unusual name because Latin was mixed up with Italian.

Mathematical formulation:

$$y^2 = \frac{4a^2(2a-x)}{x}$$

Program:

```
10 FOR X=5 TO 40 STEP 2.5
20 LET Y=6*SQR ((40-X)/X)
25 LET Y1=Y+22
30 PLOT X,Y1
40 PLOT X,22-Y
50 NEXT X
```

2. The Astroid

This curve is also known as the "tetracuspid" because of its four cusps. The curve was first studied by Jean Bernoulli (1667–1748). It is a hypocycloid for which $a = 4b$ (see curve 6 below). The curve is generated by selecting a

point on the circumference of a moving circle that rolls inside a fixed circle which is four times as large.

Mathematical formulation:

$$x=a\cos^3 t,\ y=a\sin^3 t$$

Program:

```
10 FOR T=0 TO 2*PI STEP 0.15
15 LET X=COS (T)
20 LET X=20*X*X*X
25 LET Y=SIN (T)
30 LET Y=20*Y*Y*Y
40 PLOT X+32,Y+22
50 NEXT T
```

3. The Folium of Descartes

Folium is the Latin word for leaf. This curve was first studied by René Descartes (1596–1650). Its usual equation is

$$x^3 + y^3 = 3axy$$

but as this form is useless for sketching, a parametric form had to be found.

Mathematical formulation:

$$x = \frac{3at}{1+t^3},\ y = \frac{3at^2}{1+t^3}$$

Program for portion in first quadrant:

```
 10 FOR K=1 TO 2
 20 IF K=1 THEN GOTO 50
 30 FOR T=1 TO 20 STEP 0.5
 40 GOTO 60
 50 FOR T=0 TO 1 STEP 0.1
 60 LET B=40/(1+T**3)
 70 LET X=B*T+20
 80 LET Y=B*T*T+18
 90 PLOT X,Y
100 NEXT T
110 NEXT K
```

(The parts for which $-1<t<0$ and $-\infty<t<-1$ need separate treatment.)

4. The Cycloid

The cycloid is one of the most well-known curves. It describes the motion of a point on the circumference of a rolling circle along a given straight line. It has many interesting physical applications. Galileo Galilei (1564–1642) suggested the cycloidal shape for arches of bridges.

Mathematical formulation:

$$x=a(t-\sin t),\ y=a(1-\cos t)$$

Program for one arch:

```
10 FOR T=0 TO 2*PI STEP 0.3
20 LET X=10*(T-SIN (T))
30 LET Y=10*(1-COS (T))
40 PLOT X,Y
50 NEXT T
```

5. The Epicycloid

The epicycloid describes the motion of a point on the circumference of a rolling circle on the outside of a fixed circle. (It will be a closed curve if the ratio of the fixed radius for the inner circle to the radius for the rolling outer circle is a positive integer.)

Mathematical formulation:

$$x=(a+b)\cos t-b\cos(a+b)t/b,$$
$$y=(a+b)\sin t-b\sin(a+b)t/b$$

Program for $a=10$, $b=5$, for example:

```
10 FOR T=0 TO 2*PI STEP 0.1
20 LET X=15*COS T-5*COS (3*T)
30 LET Y=15*SIN T-5*SIN (3*T)
40 PLOT X+32,Y+22
50 NEXT T
```

6. The Hypocycloid

The hypocycloid describes the motion of a point on the circumference of a rolling circle on the inside of a fixed circle.

Mathematical formulation:

$$x=(a-b)\cos t + b\cos(a-b)t/b,$$
$$y=(a-b)\sin t - b\sin(a-b)\,t/b$$

Program for $a=12, b=3$, for example:

```
  10 FOR T=0 TO 2*PI STEP 0.2
  20 LET X=13.5*COS T+4.5*COS (3
*T)
  30 LET Y=13.5*SIN T-4.5*SIN (3
*T)
  40 PLOT X+30,Y+22
  50 NEXT T
```

which happens to be the astroid again.

7. Lissajous Curves

These are also known as Bowditch Curves. Here x represents simple harmonic motion in one direction whilst y represents a different simple harmonic motion at right angles to it. Jules-Antoine Lissajous studied these curves in great detail in 1857–8. Nathaniel Bowditch found these curves useful in his book on navigation.

Mathematical formulation:

$$x=a\sin(pt/q + c),$$
$$y= b\sin t$$

Program for one with $p=2, q=3$:

```
10 FOR T=0 TO 6*PI STEP 0.2
20 LET X=20*SIN (2*T/3)
30 LET Y=20*SIN T
40 PLOT X+32,Y+22
50 NEXT T
```

8. The Epitrochoid

The epitrochoid is a first cousin to the epicycloid. It describes the motion of a point on a rolling circle (but here the point may be anywhere except on its circumference) on the outside of a fixed circle.

Mathematical formulation:

$$x=(a+b)\cos t - h\cos(a+b)t/b,$$
$$y=(a+b)\sin t - h\sin(a+b)t/b,\quad h \neq b.$$

Program for $a=23{\cdot}4, b=-31{\cdot}2, h=-10{\cdot}4$

```
   10 FOR T=0 TO 8*PI STEP 0.4
   20 LET X=10.4*COS (T/4)-7.8*CO
S  T
   30 LET Y=10.4*SIN (T/4)-7.4*SI
N  T
   40 PLOT X+32,Y+22
   50 NEXT T
```

9. The Involute of a Circle

This curve is obtained by holding the endpoint of a thin string tautly as you unwind it from a fixed circular spool. The set of all the endpoints makes up the involute.

Mathematical formulation:

$$x=a(\cos t + t\sin t),\quad y=a(\sin t - t\cos t)$$

Program:

```
10 FOR T=0 TO 8.5 STEP 0.25
20 LET X=3.5*COS T+3.5*T*SIN T
30 LET Y=3.5*SIN T-3.5*T*COS T
40 PLOT X+32,Y+22
50 NEXT T
```

10. The Nephroid

The nephroid (i.e. the kidney curve) belongs to the epicycloid family. Here the ratio of the radius to the fixed circle to that of the rolling circle is 2:1.

Mathematical formulation:

$x = a(3 \cos t - \cos 3t)$,
$y = a(3 \sin t - \sin 3t)$

Program:

```
10 FOR T=0 TO 2*PI STEP 0.2
20 LET X=5*(3*COS T-COS (3*T))
30 LET Y=5*(3*SIN T-SIN (3*T))
40 PLOT X+32,Y+22
50 NEXT T
```

11. The Deltoid

The deltoid (i.e. curve resembling a Greek delta) belongs to the hypocycloid family where the ratio of the fixed circle to that of the rolling circle is 3:1.

Mathematical formulation:

$x = a(\cos 2t + 2 \cos t)$, $y = a(2 \sin t - \sin 2t)$

Program:

```
10 FOR T=0 TO 2*PI STEP 0.2
20 LET X=7*(COS (2*T)+2*COS T)
30 LET Y=7*(2*SIN T-SIN (2*T))
40 PLOT X+32,Y+22
50 NEXT T
```

12. The Cissoid of Diocles

The cissoid (i.e. ivy-like curve) was studied by Diocles (about 100 B.C.) who used it to find a graphic solution for the classic Greek problem of the duplication of the cube. (Algebraically this means for a given 'a' find x such that $x^3 = 2a^3$.)

Mathematical formulation:

$$x = 2at^2/(1+t^2),\ y = 2at^3/(1+t^2)$$

Program:

```
10 FOR T=-1 TO 1 STEP 0.025
20 LET X=60*T*T/(1+T*T)
30 LET Y=2*X*T/3
40 PLOT X,Y+22
50 NEXT T
```

13. The Semi-Cubical Parabola

This curve owes its fame to the fact that it was the first algebraic curve whose arc length was computed, in 1659 by Neil, before the invention of the calculus.

Mathematical formulation:

$$x = 3at^2,\ y = 2at^3$$

Program:

```
10 FOR T=-1 TO 1 STEP 0.025
20 LET X=30*T*T
30 LET Y=2*X*T/3
40 PLOT X,Y+22
50 NEXT T
```

14. The Tractrix

This curve is formed when we drag a heavy particle P (which is any point on this curve) along a rough horizontal plane by a light string PB, where the end B is forced to move in a straight line which does not pass through P.

Mathematical formulation:

$x = a\{\log(\sec t + \tan t) - \sin t\}$, $y = a \cos t$.

Program:

```
10 FOR T=-1.2 TO 1.3 STEP 0.1
20 LET X=40*(LN ((1+SIN T)/COS
T)-SIN T)
30 LET Y=40*COS T
40 PLOT X+32,Y
50 NEXT T
```

15. The Spiral of Archimedes

Archimedes (about 225 B.C.) studied this curve to obtain a graphic solution to the classic problem of squaring the circle. (Algebraically that means for a given radius a of a circle find x for which $x^2 = a^2\pi$.)

Mathematical formulation:

$r = \theta$, in polar coordinates.

Program:

```
10 FOR T=-7 TO 7.2 STEP 0.3
20 LET R=4*T
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

16. Fermat's Spiral

Mathematical formulation:

$$r^2 = a^2\theta$$

Program:

```
10 FOR T=0 TO 10 STEP 0.3
20 LET R=5*SQR T
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 PLOT 32-X,22-Y
70 NEXT T
```

17. The Lemniscate of Bernoulli

First studied by Jacques Bernoulli (1654–1705), this curve belongs to the ovals of Cassini. Its main interest belongs to advanced mathematical analysis. Investigations into its arclength laid the foundations of the work on elliptic functions.

Mathematical formulation:

$$r^2 = a^2 \cos 2\theta$$

Program:

```
10 FOR T=-PI/4 TO PI/4 STEP 0.07
20 LET R=30*SQR (COS (2*T))
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 PLOT 32-X,22-Y
70 NEXT T
```

18. The Limaçon

This was first studied by Etienne Pascal (father of Blaise). It owes its name to the fact that when $a < b$ the curve resembles a snail. (The French for snail is *limaçon*.)

Mathematical formulation:

$$r = a + b \cos \theta$$

Program 1: $a < b$

```
10 FOR T=0 TO 2*PI STEP 0.2
20 LET R=12.6+21*COS T
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+10,Y+22
60 NEXT T
```

Program 2: $a > b$

```
10 FOR T=0 TO 2*PI STEP 0.2
20 LET R=15+9*COS T
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+20,Y+22
60 NEXT T
```

19. The Cardioid

This is a limaçon for which $a = b$.

Mathematical formulation:

$$r = a(1 + \cos \theta)$$

Program:

```
10 FOR T=0 TO 2*PI STEP 0.2
20 LET R=15.3*(1+COS T)
30 LET X=INT (R*COS T)
40 LET Y=INT (R*SIN T)
50 PLOT X+20,Y+22
60 NEXT T
```

20. Rose Curves

Mathematical formulation:

$$r = a \sin n\theta$$

Program 1: n an even integer

```
10 FOR T=0 TO 2*PI STEP 0.08
20 LET R=22*SIN (4*T)
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

Program 2: n an odd integer

```
10 FOR T=0 TO PI STEP 0.08
20 LET R=22*SIN (3*T)
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

21. The Strophoid

Mathematical formulation:

$$r = (a \cos 2\theta)/\cos \theta$$

Program:

```
10 FOR T=-1.2 TO 1.2 STEP 0.15
20 LET R=10*COS (2*T)/COS T
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

22. The Trisectrix of Maclaurin

This curve was first studied by C. Maclaurin (1698–1746). He used it to obtain a graphical solution to the classic problem of the trisection of an angle.

Mathematical formulation:

$$r = a \sec(\theta/3)$$

Program:

```
10 FOR T=-4.2 TO 4.2 STEP 0.36
20 LET R=4/COS (T/3)
30 LET X=2*R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

23. The Lemniscate of Gerono

This is also called the eight curve. It bears a remarkable resemblance to Bernoulli's lemniscate.

Mathematical formulation:

$$r^2 = a^2 \cos 2\theta \cos^4 \theta$$

Program:

```
  10 FOR T=-PI/4 TO PI/4 STEP 0.
1
  20 LET R=30*COS (T)*COS (T)
  30 LET R=R*SQR COS (2*T)
  40 LET X=R*COS T
  50 LET Y=R*SIN T
  60 PLOT X+32,Y+22
  70 PLOT 32-X,22-Y
  80 NEXT T
```

24. The Conchoid

This curve was first studied by the Greek mathematician Nicomedes about 225 B.C. He used it to find a graphical solution to the classic problem of the duplication of the cube. (See curve 12 above.) Note that the curve consists of two branches.

Mathematical formulation:

$$r = a + b \sec \theta$$

Program 1: $a > b$

```
 10 FOR T=-1.2 TO 1.2 STEP 0.15
 20 GOSUB 200
 30 NEXT T
 40 FOR T=1.7 TO 4.5 STEP 0.1
 50 GOSUB 200
 60 NEXT T
 70 GOTO 250
200 LET R=7.5+4.5/COS T
210 LET X=2*R*COS T
220 LET Y=R*SIN T
230 PLOT X+20,Y+22
240 RETURN
```

Program 2: $a=b$

```
 10 FOR T=-1.2 TO 1.2 STEP 0.15
 20 GOSUB 200
 30 NEXT T
 40 FOR T=1.3 TO 4.5 STEP 0.1
 50 GOSUB 200
 60 NEXT T
 70 GOTO 250
200 LET R=4.5+4.5/COS T
210 LET X=2*R*COS T
220 LET Y=R*SIN T
230 PLOT X+20,Y+22
240 RETURN
```

Program 3: $a < b$

```
 10 FOR T=-1.2 TO 1.2 STEP 0.25
 20 GOSUB 200
 30 NEXT T
 40 FOR T=1.3 TO 4.5 STEP 0.12
 50 GOSUB 200
```

```
 60 NEXT T
 70 GOTO 250
200 LET R=3+4.5/COS T
210 LET X=2*R*COS T
220 LET Y=R*SIN T
230 PLOT X+20,Y+22
240 RETURN
```

25. A Rose Relation

Mathematical formulation:

$r = a \cos(3\theta/5)$

Program:

```
10 FOR T=0 TO 15.6 STEP 0.2
20 LET R=21*COS (3*T/5)
30 LET X=R*COS T
40 LET Y=R*SIN T
50 PLOT X+32,Y+22
60 NEXT T
```

Mathematical Appendix

1. Solution of two linear equations in two unknowns

Given $ax + by = c$
and $dx + ey = f$ where $ae - bd \neq 0$

and if the values of a, b, c, d, e, f are given, then

$$x = \frac{ce - bf}{ae - bd}, \quad y = \frac{af - cd}{ae - bd}.$$

Illustration: Given:

$3x - 5y = -4$
$2x + 3y = 29$ find x and y.

Using the quoted formula above, then

$$x = \frac{-12 + 145}{9 + 10} = \frac{133}{19} = 7, \text{ and}$$

$$y = \frac{87 + 8}{9 + 10} = \frac{95}{19} = 5$$

2. Equations of straight lines

We shall only study two cases here.

Case 1:

The point-slope formula of a straight line is

$$y - y_1 = m(x - x_1)$$

Here m represents the slope of the straight line with the positive x axis and if the angle of this straight line with the x axis is α, then $m = \tan \alpha$.

Illustration: Given that the two points (3,7) and (5,4) lie on a straight line, find its equation.

$$\text{Here } m = \tan \alpha = \frac{4 - 7}{5 - 3} = -\frac{3}{2}$$

Using the point slope formula given above, then

$$y - 7 = -\frac{3}{2}(x - 3) \text{ using the first point.}$$

(Had we taken the second point we would get

$$y - 4 = -\frac{3}{2}(x - 5)$$

which amounts to the same thing.)

Case 2:

The parametric form of a straight line

$$x = a + t,\ y = b + mt.$$

Here (a,b) is a given point on the straight line and m is its slope.

Illustration: Find the parametric form for the illustration in case A above.

$$\text{Here } m = \frac{4 - 7}{5 - 3} = -\frac{3}{2} \text{ and } a = 3,\ b = 7 \text{ will do.}$$

Substituting these values

$$x = 3 + t,\ y = 7 - \frac{3t}{2}$$

(Had we taken $a = 5$, $b = 4$ we would arrive at

$$x = 5 + t,\ y = 4 - \frac{3t}{2}$$

which amounts to the same thing.)

3. Parabolas

We need study only two cases here.

Case 1: Parabolas where the axis of symmetry is parallel with the y axis

This parabola takes the form

$$y = ax^2 + bx + c$$

Illustration: Find a, b, c, of the above parabola if it passes through (1,12), (2,30), (3,58).

As the point (1,12) lies on it, we need

$$a + b + c = 12 \tag{1}$$

Now the point (2,30) also lies on it, so we need

$$4a + 2b + c = 30 \tag{2}$$

Similarly, for point (3,58) we demand

$$9a + 3b + c = 58 \tag{3}$$

Let us now subtract equation (1) from (2)

$$3a + b = 18 \tag{4}$$

Also subtract equation (1) from (3)

$$8a + 2b = 46 \tag{5}$$

We may now solve (4) and (5) using the formulae quoted in section 1.

$$a = \frac{36 - 46}{6 - 8} = 5, \; b = \frac{138 - 144}{6 - 8} = 3$$

c is found from any of the equations (1), (2) or (3).

Using equation (1),

$$c = 12 - 5 - 3 = 4$$

so the parabola has the form

$$y = 5x^2 + 3x + 4$$

Case 2: Parabolas where the axis of symmetry is parallel with the x *axis*

This parabola takes the form

$$x = ay^2 + by + c.$$

Illustration: Find a, b, c of the above parabola if it passes through (11,1), (17,2), (25,3).
As the point (11,1) lies on it we need

$$a + b + c = 11 \tag{1}$$

Similarly for point (17, 2) we need

$$4a + 2b + c = 17 \tag{2}$$

and for (25, 3) we need

$$9a + 3b + c = 25 \tag{3}$$

Let us now subtract equation (1) from (2)

$$3a + b = 6 \tag{4}$$

Also subtract equation (1) from (3)

$$8a + 2b = 14 \tag{5}$$

We may now solve equations (4) and (5) as before

$$a = \frac{12 - 14}{6 - 8} = 1, \; b = \frac{42 - 48}{6 - 8} = 3$$

Finally c may be found from equation (1)

$$c = 11 - 1 - 3 = 7$$

so the parabola has the form

$$x = y^2 + 3y + 7.$$

4. Circles

The equation to a circle centred at (a,b) with radius r is

$$(x - a)^2 + (y - b)^2 = r^2.$$

Illustration: Find the equation to the circle centred at (3, 7) with radius 4 making (i) y the dependent variable,
(ii) x the dependent variable.
The equation to this circle is:

$$(x - 3)^2 + (y - 7)^2 = 16$$

Now for case (i) $(y - 7)^2 = 16 - (x - 3)^2$

Hence $y = 7 \pm \sqrt{7 + 6x - x^2}$

i.e. $y = 7 - \sqrt{7 + 6x - x^2}$ for the branch with $y \leqslant 7$

and $y = 7 + \sqrt{7 + 6x - x^2}$ for the branch with $y \geqslant 7$

For case (ii) we have $(x - 3)^2 = 16 - (y - 7)^2$

Hence $x = 3 \pm \sqrt{14y - y^2 - 33}$

i.e. $x = 3 - \sqrt{14y - y^2 - 33}$ for the branch with $x \leqslant 3$

and $x = 3 + \sqrt{14y - y^2 - 33}$ for the branch with $x \geqslant 3$.

Solutions

Chapter 1

1. Suppose your initials are A.B.C. Then

```
 5 REM "1.1"
10 PRINT "A.B.C."
20 GOTO 10
```

2. Assume your initials are as above. Then

```
 5 REM "1.2"
10 PRINT "A.B.C.";
20 GOTO 10
```

3.

```
 5 REM "1.3"
10 PRINT "A.B.C.";TAB 26;"A.B.
C.";
20 GOTO 10
```

4.

```
 5 REM "1.4"
10 PRINT "A.                B.
      C."
20 GOTO 10
```

5.

```
 5 REM 1.5"
10 PRINT CHR$ 166;CHR$ 155;CHR
$ 167;CHR$ 155;CHR$ 166;CHR$ 155
;
20 GOTO 10
```

6.

```
 5 REM "1.6"
10 LET A=INT (63*RND)
20 IF A<38 THEN GOTO 40
30 PRINT CHR$ A;
40 GOTO 10
```

7.

```
 5 REM "1.7"
10 LET A=INT (37*RND)
20 IF A<28 THEN GOTO 40
30 PRINT CHR$ A;
40 GOTO 10
```

8.

```
 5 REM "1.8"
10 LET A=INT (165*RND)
20 IF A<156 THEN GOTO 40
30 PRINT CHR$ A;
40 GOTO 10
```

9.

```
 5 REM "1.9"
10 PRINT CHR$ 129;CHR$ 130;CHR
$ 131;CHR$ 132;CHR$ 133;CHR$ 134
;CHR$ 135;
20 GOTO 10
```

10.

```
 5 REM "1.10"
10 LET A=INT (135*RND)
20 IF A<129 THEN GOTO 40
30 PRINT CHR$ A;
40 GOTO 10
```

11.

```
 5 REM "1.11"
10 LET A=INT (21*RND)
20 LET B=INT (31*RND)
30 PRINT AT A,B;"*"
40 GOTO 10
```

12.

```
 5 REM "1.12"
10 LET A=37
20 LET A=A+1
30 IF A>63 THEN GOTO 10
40 PRINT CHR$ A;
50 GOTO 20
```

13. The solution is the same as for problem 12, but replace line 40 by

```
 5 REM "1.13"
40 PRINT CHR$ A,
```

14.

```
 5 REM "1.14"
10 PRINT CHR$ 133;"
";CHR$ 133;"                    ";CHR
$ 133
20 GOTO 10
```

15.

```
  5 REM "1.15"
 10 LET K=0
 20 LET L=0
 30 PRINT AT K,L;"*"
 40 LET K=K+1
 50 LET L=L+1
 60 IF K>21 THEN GOTO 80
 70 GOTO 30
 80 LET K=0
 90 LET L=21
100 PRINT AT K,L;"*"
110 LET L=L-1
120 LET K=K+1
130 IF K>21 THEN GOTO 150
140 GOTO 100
```

16. The solution is the same as for problem 15, replacing lines 20 and 90 with the following:

```
  5 REM "1.16"
 20 LET L=5
 90 LET L=26
```

Chapter 2

1.

```
  5 REM "2.1"
 10 LET K=63
 20 PLOT K,22
 30 UNPLOT K,22
 40 LET K=K-1
 50 IF K<0 THEN GOTO 61
 60 GOTO 20
```

2.

```
  5 REM "2.2"
 10 FOR X=0 TO 63
 20 PLOT X,11*X/16
 30 UNPLOT X,11*X/16
 40 NEXT X
 50 FOR X=63 TO 0 STEP -1
 60 PLOT X,11*X/16
 70 UNPLOT X,11*X/16
 80 NEXT X
```

3.

```
  5 REM "2.3"
 10 LET K=1
 20 FOR X=0 TO 32
 30 LET K=K+1
 40 IF K/2=INT (K/2) THEN GOTO
100
 50 FOR Y=43 TO 0 STEP -1
 60 PLOT X,Y
 70 UNPLOT X,Y
 80 NEXT Y
 90 GOTO 150
100 FOR Y=0 TO 43
110 PLOT X,Y
120 IF X=32 AND Y=22 THEN GOTO
151
130 UNPLOT X,Y
140 NEXT Y
150 NEXT X
```

4. Use the solution to problem 3 replacing lines 50 and 100 with:

```
  5 REM "2.4"
 50 FOR Y=43 TO 0 STEP -2
100 FOR Y=0 TO 43 STEP 2
```

5.

```
  5 REM "2.5"
 10 FOR K=1 TO 50
 20 LET A=INT (63*RND)
 30 LET B=INT (43*RND)
 40 PLOT A,B
 50 NEXT K
```

6.

```
  5 REM "2.6"
 10 FOR K=1 TO 50
 20 LET A=INT (63*RND)
 30 LET B=INT (43*RND)
 40 PLOT A,B
 50 IF A>=30 AND A<=34 AND B>=2
0 AND B<=24 THEN GOTO 90
 60 UNPLOT A,B
 70 NEXT K
 80 GOTO 91
 90 PRINT AT 16,9;"BRAVO.BULLSE
YE."
```

7.

```
 5 REM "2.7"
10 FOR K=0 TO 43
20 PLOT 0,K
30 UNPLOT 0,K
40 NEXT K
50 FOR K=0 TO 63
60 PLOT K,43
70 IF K=63 THEN GOTO 91
80 UNPLOT K,43
90 NEXT K
```

8.

```
 5 REM "2.8"
10 PRINT "   X              Y"
20 PRINT ,,
30 FOR X=0 TO 16
40 LET Y=X*X-20*X+105
50 PRINT "    ";X;"              ";Y
60 NEXT X
```

9.

```
 5 REM "2.9"
10 FOR X=4 TO 16
20 PLOT X,X*X-20*X+105
30 NEXT X
```

10.

```
 5 REM "2.10"
10 PRINT "   X                   Y"
20 PRINT ,,
30 FOR X=-10 TO 10
40 LET Y=40/(X*X+1)
50 PRINT "    ";X;"              ";Y
60 NEXT X
```

11.

```
 5 REM "2.11"
10 FOR X=-3 TO 3 STEP 0.1
20 LET Z=32+10*X
30 LET D=1+X*X
40 PLOT Z,40/D
50 NEXT X
```

12. (a)

```
 5 REM "2.12 A"
10 PRINT "   X                    Y"
20 PRINT ,,
30 FOR X=0 TO 60
40 LET Y=60*X/(X*X+1)
50 PRINT "    ";X;"              ";Y
60 NEXT X
```

(b)

```
 5 REM "2.12 B"
10 FOR X=0 TO 10 STEP 0.16667
20 LET Z=6*X
30 LET Y=60*X/(X*X+1)
40 PLOT Z,Y
50 NEXT X
```

(c)

```
 5 REM "2.12 C"
10 FOR X=-5 TO 5 STEP 0.08333
20 LET Z=6*X
30 LET Y=30*X/(X*X+1)
40 PLOT 32+Z,22+Y
50 NEXT X
```

13. Delete line 120 and replace:

```
 5 REM "2.13"
20 FOR X=0 TO 63
```

Insert the following two lines:

```
  65 IF X=63 AND Y=0 THEN GOTO 1
60
 160 PRINT AT 6,2;"GOSH.WHAT A S
WEAT FOR A MEASLY     PIECE OF M
EAT."
```

14.

```
  5 REM "2.14"
 10 LET A=0
 20 LET B=43
 30 LET C=1
 35 LET F=0
 40 GOSUB 150
 50 LET A=63
 55 LET E=43
 60 GOSUB 200
 70 LET A=43
 80 LET B=0
 90 LET C=-1
 95 LET F=63
100 GOSUB 150
110 LET A=63
120 LET E=0
130 GOSUB 200
150 FOR K=A TO B STEP C
160 PLOT F,K
170 UNPLOT F,K
180 NEXT K
190 RETURN
200 FOR K=A TO B STEP C
210 PLOT K,E
220 IF K=0 AND B=0 THEN GOTO 260
230 UNPLOT K,E
240 NEXT K
250 RETURN
260 PRINT AT 6,2;"HOME AT LAST."
```

Chapter 3

Corresponding program for a vertical line, using the letter grid:

```
10 FOR K=0 TO 21
20 PRINT AT K,16;CHR$ 128
30 NEXT K
```

Corresponding program for a vertical line using the plot grid:

```
10 FOR K=0 TO 43
20 PLOT 32,K
30 NEXT K
```

Corresponding program for five vertical straight lines using the letter grid:

```
10 FOR R=0 TO 28 STEP 7
20 FOR K=0 TO 21
30 PRINT AT K,R;CHR$ 128
40 NEXT K
50 NEXT R
```

and using the plot grid:

```
10 FOR R=0 TO 60 STEP 15
20 FOR K=0 TO 43
30 PLOT R,K
40 NEXT K
50 NEXT R
```

The program for the line joining (1,1) to (21,21) with slope of -1 is:

```
10 FOR X=1 TO 21
20 PRINT AT X,X;CHR$ 134
30 NEXT X
```

To write the program for the oblique straight line with slope other than $+1$ or -1 using method B, proceed as follows: First find a and b. We already know from Method A that the slope $k=½$. Let us start the straight line at $t=0$. This gives $3=a+0$ and $2=b+0$, so $a=3$ and $b=2$. Next, we need to decide how far t should go. We need $57=3+t$ so $t=54$ and we are on our way:

```
10 FOR T=0 TO 54 STEP 2
20 LET X=3+T
30 LET Y=2+T/2
40 PLOT X,Y
50 NEXT T
```

1.

```
 5 REM "3.1"
10 FOR Y=15 TO 30 STEP 15
20 FOR X=0 TO 63 STEP 2
```

```
 30 PLOT X,Y
 40 NEXT X
 50 NEXT Y
 60 FOR X=21 TO 42 STEP 21
 70 FOR Y=43 TO 0 STEP -2
 80 PLOT X,Y
 90 NEXT Y
100 NEXT X
```

2.

```
  5 REM "3.2"
 10 FOR X=7 TO 14 STEP 7
 20 FOR Y=0 TO 30 STEP 2
 30 PRINT AT X,Y;CHR$ 128;
 40 NEXT Y
 50 NEXT X
 60 FOR Y=11 TO 21 STEP 10
 70 FOR X=0 TO 20 STEP 2
 80 PRINT AT X,Y;CHR$ 128
 90 NEXT X
100 NEXT Y
```

3. We first need to realize that the straight line from A(2,5) to B(22,30) may be expressed as

$$x=2+t \text{ and } y=5+5t/4 \text{ with } 0\leqslant t\leqslant 20$$

whilst the straight line joining B(22,30) to C (52,5) may be represented as

$$x=22+t \text{ and } y=30 - 5t/6 \text{ with } 0\leqslant t\leqslant 30.$$

Then we program as follows:

```
  5 REM "3.3"
 10 FOR T=0 TO 20 STEP 2
 20 LET X=2+T
 30 LET Y=5+5*T/4
 40 PLOT X,Y
 50 NEXT T
 60 FOR T=0 TO 30 STEP 2
 70 LET X=22+T
 80 LET Y=30-5*T/6
 90 PLOT X,Y
100 NEXT T
```

4. The line segment between A(1,1) and B(16,31) may be written as

   $x=1+t$, $y=1+2t$ for $0 \leqslant t \leqslant 15$.

   The line segment between B(16,31) and C(41,6) is then

   $x=16+t$, $y=31-t$ for $0 \leqslant t \leqslant 25$

   and the last line segment linking C(41,6) with D(59,42) will be

   $x=41+t$, $y=6+2t$ for $0 \leqslant t \leqslant 18$

   So we program as follows:

```
  5 REM "3.4"
 10 FOR T=0 TO 15 STEP 2
 20 LET X=1+T
 30 LET Y=1+2*T
 40 PLOT X,Y
 50 NEXT T
 60 FOR T=0 TO 25 STEP 2
 70 LET X=16+T
 80 LET Y=31-T
 90 PLOT X,Y
100 NEXT T
110 FOR T=0 TO 18 STEP 2
120 LET X=41+T
130 LET Y=6+2*T
140 PLOT X,Y
150 NEXT T
```

5. For the first leg of the W let us join A(0,42) to B(15,0). The relevant straight line segment is:

   $x=t$, $y=42-14t/5$; $0 \leqslant t \leqslant 15$.

   For the second leg we join B(15,0) to C(30,42) and arrive at

   $x=15+t$, $y=14t/5$; $0 \leqslant t \leqslant 15$.

   For the third leg we join C(30,42) to D(45,0) and get

   $x=30+t$, $y=42-14t/5$; $0 \leqslant t \leqslant 15$.

For the last leg we join D(45,0) to E(60,42) and find:

$x=45+t$, $y=14t/5$; $0 \leqslant t \leqslant 15$.

So we program as follows:

```
  5 REM "3.5"
 10 FOR T=0 TO 15 STEP 2
 20 LET X=T
 30 LET Y=42-14*T/5
 40 PLOT X,Y
 50 NEXT T
 60 FOR T=0 TO 15 STEP 2
 70 LET X=15+T
 80 LET Y=14*T/5
 90 PLOT X,Y
100 NEXT T
110 FOR T=0 TO 15 STEP 2
120 LET X=30+T
130 LET Y=42-14*T/5
140 PLOT X,Y
150 NEXT T
160 FOR T=0 TO 15 STEP 2
170 LET X=45+T
180 LET Y=14*T/5
190 PLOT X,Y
200 NEXT T
```

6. Here the neatest solution is to draw four parallel straight lines of slope 14/5 and another set of four parallel straight lines of slope $-14/5$.
   For the parallel lines with positive slope we program:

```
  5 REM "3.6"
 10 FOR A=0 TO 45 STEP 15
 20 FOR Y=0 TO 42 STEP 2
 30 PLOT (5*Y+14*A)/14,Y
 40 NEXT Y
 50 NEXT A
```

whilst for those with negative slope we program:

```
 60 FOR B=15 TO 60 STEP 15
 70 FOR Y=0 TO 42 STEP 2
 80 PLOT (14*B-5*Y)/14,Y
 90 NEXT Y
100 NEXT B
```

7. Let us first shade in the two parts where the mountains have a positive slope:

```
  4 REM "3.7"
  5 LET K=0
 10 FOR A=0 TO 30 STEP 30
 20 LET K=K+1
 30 IF K=1 THEN GOTO 70
 40 LET B=30
 50 LET C=45
 60 GOTO 90
 70 LET B=0
 80 LET C=15
 90 FOR X=B TO C
100 LET Y=INT (14*(X-A)/5)
110 FOR Z=0 TO Y
120 PLOT X,Z
130 NEXT Z
140 NEXT X
150 NEXT A
```

This is followed by shading in the two parts where the mountains have a negative slope:

```
160 FOR B=30 TO 60 STEP 30
170 LET K=K+1
180 IF K=3 THEN GOTO 220
190 LET D=46
200 LET E=60
210 GOTO 240
220 LET D=16
230 LET E=30
240 FOR X=D TO E
250 LET Y=INT (14*(B-X)/5)
260 FOR Z=0 TO Y
270 PLOT X,Z
280 NEXT Z
290 NEXT X
300 NEXT B
```

8. (The 16K RAM pack is not needed here.)

```
 5 REM "3.8"
10 FOR X=0 TO 60
20 LET Y=INT (2*X-X*X/30)
30 FOR Z=0 TO Y
40 PLOT X,Z
50 NEXT Z
60 NEXT X
```

9. (The 16K RAM pack is not needed.)

```
  5 REM "3.9"
 10 LET K=0
 20 FOR X=0 TO 60
 30 LET Y=INT (2*X-X*X/30)
 40 LET K=K+1
 50 IF K/2=INT (K/2) THEN GOTO
80
 60 LET A=1
 70 GOTO 90
 80 LET A=0
 90 FOR Z=A TO Y STEP 2
100 PLOT X,Z
110 NEXT Z
120 NEXT X
```

10.

```
  5 REM "3.10"
 10 FOR X=0 TO 62 STEP 31
 20 FOR Y=0 TO 42
 30 PLOT X,Y
 40 NEXT Y
 50 NEXT X
 60 LET B=0
 70 GOSUB 110
 80 LET B=31
 90 GOSUB 110
100 GOTO 151
110 FOR Z=0 TO 31 STEP 2
120 LET Y=42+2*Z*(Z-31)/45
130 PLOT Z+B,Y
140 NEXT Z
150 RETURN
```

11. Use all programming steps from problem 10 and add the following:

```
155 REM "3.11"
160 LET K=1
170 FOR X=7 TO 23
180 LET Y=INT (42+2*X*(X-31)/45

190 LET K=K+1
200 IF K/2=INT (K/2) THEN GOTO
230
210 LET A=11
220 GOTO 240
230 LET A=10
240 FOR Z=A TO Y STEP 2
250 PLOT X,Z
260 NEXT Z
270 NEXT X
```

12.

```
  5 REM "3.12"
 10 FOR X=16 TO 44 STEP 28
 20 FOR Y=0 TO 25
 30 PLOT X,Y
 40 NEXT Y
 50 NEXT X
 60 FOR X=15 TO 45 STEP 2
 65 LET D=(X-15)*(X-45)/225
 70 LET Y1=25+7*D
 80 LET Y2=25-7*D
 90 LET Y3=25-16*D
100 PLOT X,Y3
110 PLOT X,Y2
120 PLOT X,Y1
130 PLOT X,0
140 NEXT X
```

13. Use all the programming steps from problem 12 and add the following:

```
145 REM "3.13"
150 LET K=0
160 FOR X=16 TO 44
170 LET D=(X-15)*(X-45)/225
180 LET Y=INT (25+7*D)
190 LET K=K+1
200 IF K/2=INT (K/2) THEN GOTO
230
210 LET A=1
220 GOTO 240
230 LET A=0
240 FOR Z=A TO Y STEP 2
250 PLOT X,Z
260 NEXT Z
270 NEXT X
```

Chapter 4

The coordinates of the nine squares could be top left (3,5); top middle (3,15); top right (3,25); centre left (10,5); centre middle (10,15); centre right (10,25); bottom left (17,5); bottom middle (17,15); bottom right (17,25).

The relevant GOTO command is:

```
120 GOTO 60
```

1. The time delay of approximately one second is achieved by letting the computer do a loop doing nothing but mark time for about a second. This is done using a simple subroutine for each jump. The first five lines of the program are as before, then add:

```
 55 REM "4.1"
 60 FOR Y=8 TO 36 STEP 14
 70 PLOT 10,Y
 80 GOSUB 600
 90 UNPLOT 10,Y
100 NEXT Y
110 FOR X=31 TO 51 STEP 20
120 PLOT X,36
130 GOSUB 600
140 UNPLOT X,36
150 NEXT X
160 GOTO 621
600 FOR K=1 TO 43
610 NEXT K
620 RETURN
```

2. The first 15 lines are the same as in problem 1, then add:

```
155 REM "4.2"
160 FOR Y=22 TO 8 STEP -14
170 PLOT 51,Y
180 GOSUB 600
190 UNPLOT 51,Y
200 NEXT Y
210 FOR X=31 TO 10 STEP -21
220 PLOT X,8
230 GOSUB 600
240 UNPLOT X,8
250 NEXT X
260 GOTO 621
600 FOR K=1 TO 43
610 NEXT K
620 RETURN
```

3. The first ten lines are the same as in problem 1, then add:

```
105 REM "4.3"
110 FOR Y=36 TO 8 STEP -14
120 PLOT 31,Y
130 GOSUB 600
140 UNPLOT 31,Y
150 NEXT Y
160 FOR Y=8 TO 36 STEP 14
170 PLOT 51,Y
180 GOSUB 600
190 UNPLOT 51,Y
200 NEXT Y
210 GOTO 621
600 FOR K=1 TO 43
610 NEXT K
620 RETURN
```

Chapter 5

Relevant programming steps for the remaining 16 black squares are:

```
60 FOR X=2 TO 14 STEP 4
70 FOR Y=2 TO 14 STEP 4
80 GOSUB 100
82 NEXT Y
84 NEXT X
90 GOTO 131
```

At line 90 we could have written 90 STOP which would result in the message 9/90. But it is neater to go to the end of the program and wind up with 0/90.

To place the chessboard design in the centre of the screen replace:

```
10 FOR X=3 TO 15 STEP 4
20 FOR Y=7 TO 19 STEP 4
60 FOR X=5 TO 17 STEP 4
70 FOR Y=9 TO 21 STEP 4
```

1.

```
  4 REM "5.1"
  5 LET A$=CHR$ 128
 10 FOR X=3 TO 11 STEP 8
 20 FOR Y=7 TO 15 STEP 8
 30 GOSUB 100
 40 NEXT Y
 50 NEXT X
 60 FOR X=7 TO 15 STEP 8
 70 FOR Y=11 TO 19 STEP 8
 80 GOSUB 100
 82 NEXT Y
 84 NEXT X
 90 GOTO 131
100 FOR K=0 TO 3
110 PRINT AT X+K,Y;A$;A$;A$;A$;
120 NEXT K
130 RETURN
```

2.

```
  4 REM "5.2"
  5 LET A$=CHR$ 6
 10 FOR X=3 TO 11 STEP 8
 20 FOR Y=7 TO 15 STEP 8
 30 GOSUB 100
 40 NEXT Y
 50 NEXT X
 60 FOR X=7 TO 15 STEP 8
 70 FOR Y=11 TO 19 STEP 8
 80 GOSUB 100
 82 NEXT Y
 84 NEXT X
 90 GOTO 131
100 FOR K=0 TO 3
110 PRINT AT X+K,Y;A$;A$;A$;A$;
120 NEXT K
130 RETURN
```

3.

```
  3 REM "5.3"
  4 LET A$=CHR$ 128
  5 LET B$=CHR$ 0
 10 PRINT A$;B$;A$;
 20 GOTO 10
```

4.

```
  3 REM "5.4"
  4 LET A$=CHR$ 128
  5 LET B$=CHR$ 0
 10 PRINT A$;B$;
 20 GOTO 10
```

(In fact if you then type in 'CONT' and 'ENTER' you will get another screenful. This comment is of course also valid for problem 3 above.)

5. The first 15 lines are the same as before. Now add:

```
135 REM "5.5"
140 LET A=15
150 LET B=7
160 FOR K=0 TO 28 STEP 4
170 PLOT A+K,B+K
180 GOSUB 500
190 UNPLOT A+K,B+K
200 NEXT K
210 GOTO 521
500 FOR M=1 TO 43
510 NEXT M
520 RETURN
```

(N.B. We could alternatively use PAUSE 50 instead of the quoted subroutine but the above procedure is easier on the eyes.)

6. The first 15 lines are the same as before. We now program as follows:

```
140 REM "5.6"
160 FOR K=0 TO 24 STEP 8
170 PLOT 15,7+K
180 GOSUB 500
190 UNPLOT 15,7+K
200 NEXT K
210 FOR K=0 TO 24 STEP 8
220 PLOT 19+K,35
230 GOSUB 500
240 UNPLOT 19+K,35
250 NEXT K
260 GOTO 521
500 FOR M=1 TO 43
510 NEXT M
520 RETURN
```

7. The first 15 lines are the same as usual. Then proceed as follows:

```
135 REM "5.7"
140 FOR A=15 TO 39 STEP 8
160 FOR K=0 TO 24 STEP 8
170 PLOT A,7+K
180 GOSUB 500
190 UNPLOT A,7+K
200 NEXT K
210 FOR K=0 TO 24 STEP 8
220 PLOT A+4,35-K
230 GOSUB 500
240 UNPLOT A+4,35-K
250 NEXT K
260 NEXT A
270 GOTO 521
500 FOR M=1 TO 43
510 NEXT M
520 RETURN
```

Chapter 6

The relevant program segment for *y* is:

```
  80 IF Y>B THEN GOTO 300
  90 IF Y<B THEN GOTO 320
  95 IF Y=B THEN PRINT "YOUR Y I
S O.K."
  96 IF X=A AND Y=B THEN PRINT "
CONGRATULATIOS,YOU HAVE FOUND IT
."
  97 IF X=A AND Y=B THEN GOTO 33
1
 115 GOTO 30
 200 PRINT "TREASURE IS LEFT."
 210 GOTO 80
 250 PRINT "TREASURE IS RIGHT."
 260 GOTO 80
 300 PRINT "TREASURE IS LOWER"
 310 GOTO 30
 320 PRINT "TREASURE IS HIGHER,"
 330 GOTO 30
```

1. Set

```
  5 REM "6.1"
 10 LET A=INT (4*RND)
 20 LET B=INT (4*RND)
```

then proceed as before.

2.

```
  4 REM "6.2"
  5 LET I=0
 45 LET I=I+1
 46 PRINT "MOVE NUMBER ";I,,
```

3.

```
  5 REM "6.3"
 10 LET A=INT (21*RND)
 20 LET B=INT (31*RND)
200 PRINT "TREASURE IS HIGHER,"
250 PRINT "TREASURE IS LOWER,"
300 PRINT "TREASURE IS LEFT."
320 PRINT "TREASURE IS RIGHT."
```

4.

```
  2 REM "6.4"
331 FOR K=0 TO 21
332 PRINT ,,
334 NEXT K
335 PLOT A,B
```

You will first get the error message 5/332. If you then type 'CONT' and 'ENTER' you will see the location of the treasure on the screen.
An even neater solution is to replace line 332 by

```
332 SCROLL
```

and we obtain the location of the treasure without intermediate steps.

Chapter 7

1. Line AB is vertical, so this presents no problem. The equations to lines AC and BC are respectively:

   $y=4(95-x)/9$ and $y=2(x+40)/9$

   Program as follows:

```
 5 REM "7.1"
10 FOR X=5 TO 50 STEP 2
20 PLOT X,4*(95-X)/9
30 PLOT X,2*(X+40)/9
40 NEXT X
50 FOR Y=10 TO 40 STEP 2
60 PLOT 5,Y
70 NEXT Y
```

2. For the flagpole we write:

```
 5 REM "7.2"
10 FOR Y=0 TO 42
20 PLOT 4,Y
30 NEXT Y
```

Our next task is to darken the triangular pennant:

```
 40 FOR X=5 TO 50
 50 LET Y1=INT (2*(X+40)/9)
 60 LET Y2=INT (4*(95-X)/9)
 70 FOR Z=Y1 TO Y2
 80 PLOT X,Z
 90 NEXT Z
100 NEXT X
```

3. Here the equations to lines AB, AC and BC are respectively:

$y=70-6x$, $y=(415-3x)/10$, $y=(20+x)/3$

Program as follows:

```
  5 REM "7.3"
 10 FOR X=5 TO 10
 20 PLOT X,(415-3*X)/10
 30 PLOT X,70-6*X
 40 NEXT X
 50 FOR X=10 TO 55 STEP 2
 60 PLOT X,(415-3*X)/10
 70 PLOT X,(20+X)/3
 80 NEXT X
```

4. Add the following lines:

```
  82 REM "7.4"
  85 LET K=0
  90 FOR X=15 TO 29
 100 LET K=K+1
 110 IF K/2=INT (K/2) THEN GOTO
140
 120 LET A=18
 130 GOTO 150
 140 LET A=19
 150 FOR Z=A TO 30 STEP 2
 160 PLOT X,Z
 170 NEXT Z
 180 NEXT X
```

5. (The 16K RAM Pack is needed here.) This design could be done on either grid. Using the plot grid we have for the flagpole:

```
 5 REM "7.5"
10 FOR X=4 TO 5
20 FOR Y=0 TO 43
30 PLOT X,Y
40 NEXT Y
50 NEXT X
```

We now realize that we have two pairs of congruent parallelograms so two subroutines are indicated: one for the two rising panels and the other for the two descending ones. Program as follows:

```
 60 FOR M=0 TO 1
 70 IF M=0 THEN GOTO 100
 80 LET X=22
 90 GOTO 110
100 LET X=6
110 GOSUB 400
120 NEXT M
130 FOR N=0 TO 1
140 IF N=0 THEN GOTO 170
150 LET X=30
160 GOTO 180
170 LET X=14
180 GOSUB 600
190 NEXT N
200 GOTO 711
400 FOR Z=X TO X+7
410 IF M=0 THEN GOTO 450
420 LET A=62-Z
430 LET B=44-Z
440 GOTO 470
450 LET A=46-Z
460 LET B=28-Z
470 FOR Y=A TO B STEP -2
480 PLOT Z,Y
490 NEXT Y
500 NEXT Z
510 RETURN
600 FOR Z=X TO X+7
610 IF N=0 THEN GOTO 650
620 LET A=Z-16
630 LET B=Z+2
640 GOTO 670
650 LET A=Z
660 LET B=Z+18
670 FOR Y=A TO B STEP 2
680 PLOT Z,Y
690 NEXT Y
700 NEXT Z
710 RETURN
```

6. Let us place the three vertices of this triangle at A(11,5), B(51,5), C(31,5+20$\sqrt{3}$).
Then equations to AC and CB are respectively:

$$y=1{\cdot}732x-14{\cdot}053 \text{ and } y=93{\cdot}335-1{\cdot}732x$$

so we may program as follows for the first half:

```
  5 REM "7.6"
 10 LET K=0
 20 FOR X=11 TO 31
 30 LET K=K+1
 40 IF K/2=INT (K/2) THEN GOTO
70
 50 LET A=6
 60 GOTO 80
 70 LET A=5
 80 LET Y=1.732*X-14.053
 90 FOR Z=A TO Y STEP 2
100 PLOT X,Z
110 NEXT Z
115 NEXT X
```

For the second half we continue:

```
119 LET K=0
120 FOR X=32 TO 51
130 LET K=K+1
140 IF K/2=INT (K/2) THEN GOTO
170
150 LET A=5
160 GOTO 180
170 LET A=6
180 LET Y=93.335-1.732*X
190 FOR Z=A TO Y STEP 2
200 PLOT X,Z
210 NEXT Z
220 NEXT X
```

7. (The 16K RAM Pack is needed here.)

```
  5 REM "7.7"
 10 FOR X=54 TO 39 STEP -2
 20 PLOT X,96.322-1.376*X
 30 PLOT X,1.3763*X-52.322
 40 NEXT X
 50 FOR X=43 TO 35 STEP -2
 60 PLOT X,81.183-1.376*X
```

```
 70 PLOT X,1.376*X-37.181
 80 NEXT X
 90 FOR X=39 TO 14 STEP -2
100 PLOT X,0.3249*X+30.318
110 PLOT X,13.684-0.3249*X
120 NEXT X
130 FOR X=35 TO 23 STEP -2
140 PLOT X,20.960+0.3249*X
150 PLOT X,23.040-0.3249*X
160 NEXT X
170 FOR Y=35 TO 9 STEP -2
180 PLOT 14,Y
190 NEXT Y
200 FOR Y=28 TO 16 STEP -2
210 PLOT 23,Y
220 NEXT Y
```

8. All the above steps plus:

```
225 REM "7.8"
230 FOR X=9 TO 23
240 FOR Y=19 TO 25
250 PLOT X,Y
260 NEXT Y
270 NEXT X
280 FOR X=48 TO 53
290 LET Y1=1.3763*X-52.322
300 LET Y2=96.322-1.3763*X
310 FOR Z=Y1 TO Y2
320 PLOT X,Z
330 NEXT Z
340 NEXT X
```

9. (The 16K RAM Pack is needed here.)

```
  5 REM "7.9"
 10 FOR X=14 TO 39
 20 LET Y1=INT (13.684-0.3249*X
)
 30 LET Y2=INT (0.3249*X+30.318
)
 40 FOR Z=Y1 TO Y2
 50 PLOT X,Z
 60 NEXT Z
 70 NEXT X
 80 FOR X=39 TO 54
 90 LET Y1=INT (1.3763*X-52.322
)
100 LET Y2=INT (96.322-1.3760*X
)
110 FOR Z=Y1 TO Y2
120 PLOT X,Z
130 NEXT Z
140 NEXT X
```

10. (The 16K RAM Pack is needed here.)

```
  5 REM "7.10"
 10 FOR X=12 TO 22
 20 LET Y1=INT (42.785-1.732*X)
 30 LET Y2=INT (1.732*X+1.215)
 40 FOR Z=Y1 TO Y2
 50 PLOT X,Z
 60 NEXT Z
 70 NEXT X
 80 FOR X=23 TO 42
 90 FOR Y=4 TO 39
100 PLOT X,Y
110 NEXT Y
120 NEXT X
130 FOR X=43 TO 52
140 LET Y1=INT (1.732*X-68.067)
150 LET Y2=INT (112.067-1.732*X

160 FOR Z=Y1 TO Y2
170 PLOT X,Z
180 NEXT Z
190 NEXT X
```

11. The equation for the lower arc to the circle is $y=22-\sqrt{400-(x-32)^2}$ whilst the upper arc is $y=22+\sqrt{400-(x-32)^2}$ so we program as follows:

```
  5 REM "7.11"
 10 FOR X=12 TO 52
 20 LET D=SQR (64*X-X*X-624)
 30 LET Y1=22-D
 40 LET Y2=22+D
 50 FOR Z=Y1 TO Y2
 60 PLOT X,Z
 70 NEXT Z
 80 NEXT X
```

12. (The 16K RAM pack is needed here.) After some thought it becomes clear that this design is drawn best when using horizontal lines. It then falls into four distinct sections.

For section one we draw the horizontal lines which lie between $x=0{\cdot}5774y-0{\cdot}702$ and $x=64{\cdot}702-0{\cdot}5774y$, so we program as follows:

```
 5 REM "7.12"
10 FOR Y=39 TO 31 STEP -1
20 LET X1=0.5774*Y-0.7025
30 LET X2=64.702-0.5774*Y
40 FOR Z=X1 TO X2
50 PLOT Z,Y
60 NEXT Z
70 NEXT Y
```

In section two we handle two areas simultaneously.
One area lies between $x=0{\cdot}5774y-0{\cdot}702$
and $x=0{\cdot}5774y+9{\cdot}298$
and the other lies between
$x=54{\cdot}702-0{\cdot}5774y$
and $x=64{\cdot}702-0{\cdot}5774y$
so we continue as shown:

```
 80 FOR Y=31 TO 22 STEP -1
 90 LET X1=0.5774*Y-0.702
100 LET X2=0.5774*Y+9.298
110 FOR Z=X1 TO X2
120 PLOT Z,Y
130 NEXT Z
140 LET X3=54.702-0.5774*Y
150 LET X4=64.702-0.5774*Y
160 FOR Z=X3 TO X4
170 PLOT Z,Y
180 NEXT Z
190 NEXT Y
```

Section three also handles two areas simultaneously.
One area lies between $x=24{\cdot}702-0{\cdot}5774y$
and $x=34{\cdot}702-0{\cdot}5774y$
whilst the other area lies between
$x=29{\cdot}298+0{\cdot}5774y$
and $x=39{\cdot}299+0{\cdot}5774y$
So for section three we program as follows:

```
200 FOR Y=22 TO 13 STEP -1
210 LET X1=24.702-0.5774*Y
220 LET X2=34.702-0.5774*Y
```

```
230 FOR Z=X1 TO X2
240 PLOT Z,Y
250 NEXT Z
260 LET X3=29.298+0.5774*Y
270 LET X4=39.299+0.5774*Y
280 FOR Z=X3 TO X4
290 PLOT Z,Y
300 NEXT Z
310 NEXT Y
```

Section four, the final section, merely draws two horizontal straight lines as a base for this tunnel. No trouble there:

```
320 FOR X=8 TO 55
330 FOR Y=12 TO 11 STEP -1
340 PLOT X,Y
350 NEXT Y
360 NEXT X
```

13. (The 16K RAM Pack is needed here.) This design may be done either by drawing horizontal lines or vertical ones. We split the design into two distinct parts. First let the equation to the outer circle be

$$(x-32)^2+(y-22)^2=400$$

and that of the inner circle be

$$(x-32)^2+(y-22)^2=100$$

For the first part we fill in the left part of the ring for which $12\leqslant x\leqslant 22$ as well as the right part of the ring for which $42\leqslant x\leqslant 52$. So we program as follows:

```
  5 REM "7.13"
 10 FOR M=0 TO 1
 20 IF M=1 THEN GOTO 60
 30 LET A=12
 40 LET B=22
 50 GOTO 80
 60 LET A=42
 70 LET B=52
 80 FOR X=A TO B
 90 LET D=SQR (64*X-X*X-624)
100 LET Y1=22-D
110 LET Y2=22+D
```

```
120 FOR Z=Y1 TO Y2
130 PLOT X,Z
140 NEXT Z
150 NEXT X
160 NEXT M
```

For the second part we fill in the remaining two areas, the top area for which

$$22-\sqrt{64x-x^2-624} \leqslant y \leqslant 22-\sqrt{64x-x^2-924}$$

and $23 \leqslant x \leqslant 41$

as well as the bottom area for which

$$22+\sqrt{64x-x^2-924} \leqslant y \leqslant 22+\sqrt{64x-x^2-624}$$

and x is in the same range as above.
So we program as shown:

```
170 FOR X=23 TO 41
180 LET D=SQR (64*X-X*X-624)
190 LET E=SQR (64*X-X*X-924)
200 LET Y1=22-D
210 LET Y2=22-E
220 FOR Z=Y1 TO Y2
230 PLOT X,Z
240 NEXT Z
250 LET Y3=22+E
260 LET Y4=22+D
270 FOR Z=Y3 TO Y4
280 PLOT X,Z
290 NEXT Z
300 NEXT X
```

14. (The 16K RAM Pack is needed here.) The design falls naturally into four sections.
For section one we cover the area between

$$y=50\cdot970-2\cdot4142x$$

and $y=2\cdot4142x-6\cdot414$

with x ranging from 12 to 18.

So we program as follows:

```
  5 REM "7.14"
 10 FOR X=12 TO 18
 20 LET Y1=50.970-2.4142*X
 30 LET Y2=2.4142*X-6.970
 40 FOR Z=Y1 TO Y2
 50 PLOT X,Z
 60 NEXT Z
 70 NEXT X
```

In section two we cover the area between

$$y=15{\cdot}255-0{\cdot}4142x$$
and $y=28{\cdot}745+0{\cdot}4142x$

with x ranging from 18 to 32
and we carry on as follows:

```
 80 FOR X=18 TO 32
 90 LET Y1=15.255-0.4142*X
100 LET Y2=28.745+0.4142*X
110 FOR Z=Y1 TO Y2
120 PLOT X,Z
130 NEXT Z
140 NEXT X
```

For section three we cover the area between the lines

$$y=0{\cdot}4142x-11{\cdot}254$$
and $y=55{\cdot}255-0{\cdot}4142x$

with x ranging from 32 to 46
so we write:

```
150 FOR X=32 TO 46
160 LET Y1=0.41421*X-11.2544
170 LET Y2=55.255-0.41421*X
180 FOR Z=Y1 TO Y2
190 PLOT X,Z
200 NEXT Z
210 NEXT X
```

For the final section we cover the area between

$$y=2\cdot4142x-103\cdot538$$
and $y=147\cdot538-2\cdot4142x$

with x from 46 to 52
and the programming for the last section runs:

```
220 FOR X=47 TO 52
230 LET Y1=2.41421*X-103.5385
240 LET Y2=147.538-2.4121*X
250 FOR Z=Y1 TO Y2
260 PLOT X,Z
270 NEXT Z
280 NEXT X
```

15. All the programming steps of problem 14 plus:

```
285 REM "7.15"
290 FOR X=22 TO 42
300 FOR Y=12 TO 32
310 UNPLOT X,Y
320 NEXT Y
330 NEXT X
```

16. (The 16K RAM Pack is needed here.) Figure 7.16 is made up of three identical triangles with the base at the top, and another three identical triangles with the base at the bottom, so it seems expedient to devise two subroutines for these. Once this has been done we simply compute the starting point of each triangle and we are on our way:

```
  5 REM "7.16"
 10 LET A=3
 15 LET B=32
 20 GOSUB 400
 40 LET A=41
 50 GOSUB 400
 60 LET A=22
 70 LET B=14
 80 GOSUB 400
 90 LET B=32
100 GOSUB 300
110 LET A=3
120 LET B=14
```

```
130 GOSUB 300
140 LET A=41
150 GOSUB 300
160 GOTO 490
300 LET K=-1
310 FOR Y=B TO B+10
320 LET K=K+1
330 IF K=10 THEN GOTO 380
340 FOR X=A+K TO 18+A-K STEP 2
350 PLOT X,Y
360 NEXT X
370 NEXT Y
380 RETURN
400 LET K=-1
410 FOR Y=B TO B-10 STEP -1
420 LET K=K+1
430 IF K=10 THEN GOTO 480
440 FOR X=A+K TO 18+A-K STEP 2
450 PLOT X,Y
460 NEXT X
470 NEXT Y
480 RETURN
```

Chapter 8

1. Three subroutines are indicated: one for the three horizontal straight lines, one for the four vertical straight lines and the last for the three oblique lines.

   Stage 1: Subroutines:

```
400 FOR Z=A TO B
410 PLOT Z,Y
420 NEXT Z
430 RETURN
500 FOR Z=A TO B
510 PLOT X,Z
520 NEXT Z
530 RETURN
600 FOR Z=0 TO 3
610 PLOT A+Z,B+Z
620 NEXT Z
630 RETURN
```

Stage 2: Do the horizontal lines.

```
  5 REM "0,1"
 10 LET A=17
 20 LET B=41
 30 LET Y=40
 40 GOSUB 400
 50 LET A=12
 60 LET B=36
 70 LET Y=35
 80 GOSUB 400
 90 LET Y=6
100 GOSUB 400
```

Stage 3: Do the vertical lines.

```
110 LET A=6
120 LET B=34
130 LET X=12
140 GOSUB 500
150 LET X=24
160 GOSUB 500
170 LET X=36
180 GOSUB 500
190 LET A=11
200 LET B=39
210 LET X=41
220 GOSUB 500
```

Stage 4: Put in the two door knobs.

```
230 PLOT 22,20
240 PLOT 26,20
```

Stage 5: Do the three oblique lines.

```
250 LET A=13
260 LET B=36
270 GOSUB 600
280 LET A=37
290 GOSUB 600
300 LET B=7
310 GOSUB 600
320 GOTO 640
```

2.

Stage 1: Use the three subroutines used in problem 1.

Stage 2: Do the horizontal lines.

```
  5 REM "8.2"
 10 LET A=16
 20 LET B=39
 30 LET Y=2
 40 GOSUB 400
 50 LET Y=3
 60 GOSUB 400
 70 LET Y=11
 80 GOSUB 400
 90 LET Y=19
100 GOSUB 400
110 LET Y=27
120 GOSUB 400
130 LET A=21
140 LET B=44
150 LET Y=32
160 GOSUB 400
```

Stage 3: Do the three vertical lines.

```
170 LET A=4
180 LET B=26
190 LET X=16
200 GOSUB 500
210 LET X=39
220 GOSUB 500
230 LET A=7
240 LET B=31
250 LET X=44
260 GOSUB 500
```

Stage 4: Do the three knobs for each drawer.

```
270 FOR Y=7 TO 23 STEP 8
280 FOR X=27 TO 28
290 PLOT X,Y
300 NEXT X
310 NEXT Y
```

Stage 5: Do the three oblique lines.

```
320 LET A=17
330 LET B=28
340 GOSUB 600
350 LET A=40
360 GOSUB 600
370 LET B=3
380 GOSUB 600
390 GOTO 640
```

3. We shall need two subroutines, one for an oblique rising line and the other for an oblique descending line:

Stage 1: Subroutines:

```
500 FOR Z=0 TO 19
510 PLOT A+Z,B+Z
520 NEXT Z
530 RETURN
600 FOR Z=0 TO 19
610 PLOT A+Z,B-Z
620 NEXT Z
630 RETURN
```

Stage 2: Do the two horizontal straight lines.

```
  5 REM "8.3"
 10 FOR Y=29 TO 33
 20 FOR X=3 TO 29
 30 PLOT X,Y
 40 NEXT X
 50 NEXT Y
 60 FOR X=3 TO 29
 70 UNPLOT X,31
 80 NEXT X
```

Stage 3: Do the oblique descending lines.

```
 90 LET B=26
100 FOR A=12 TO 16
110 GOSUB 600
120 NEXT A
130 FOR Z=0 TO 19
140 UNPLOT 14+Z,26-Z
150 NEXT Z
```

Stage 4: Do the oblique rising lines.

```
160 LET B=22
170 FOR A=21 TO 25
180 GOSUB 500
190 NEXT A
200 FOR Z=0 TO 19
210 UNPLOT 23+Z,22+Z
220 NEXT Z
230 PLOT 24,21
```

Stage 5: Join the horizontal line to the oblique rising lines.

```
240 PRINT AT 5,15;CHR$ 128
250 GOTO 640
```

4. Most of this design can be drawn on the letter grid. One subroutine is indicated for the many horizontal lines. A suitable one is listed below.

Stage 1: Subroutine:

```
500 FOR X=A TO B
510 FOR Y=C TO D
520 PRINT AT X,Y;CHR$ 128
530 NEXT Y
540 NEXT X
550 RETURN
```

Stage 2: Draw the horizontal lines (top half).

```
  5 REM "8.4"
 10 LET A=0
 20 LET B=10
 30 LET C=14
 40 LET D=31
 50 GOSUB 500
 60 FOR X=2 TO 6 STEP 3
 70 FOR Y=14 TO 31
 80 PRINT AT X,Y;CHR$ 0
 90 NEXT Y
100 NEXT X
```

Stage 3: Draw the horizontal lines (bottom half).

```
110 LET A=12
120 LET B=19
130 LET C=2
140 GOSUB 500
150 FOR X=14 TO 17 STEP 3
160 FOR Y=2 TO 31
170 PRINT AT X,Y;CHR$ 0
180 NEXT Y
190 NEXT X
```

Stage 4: Do all the stars.

```
195 FOR K=0 TO 1
200 FOR X=1+K TO 11-K STEP 2
210 FOR Y=2+K TO 12-K STEP 2
220 PRINT AT X,Y;CHR$ 23
230 NEXT Y
240 NEXT X
250 NEXT K
```

Stage 5: Do the flagpole.

```
260 FOR Y=0 TO 43
270 PLOT 3,Y
280 NEXT Y
290 GOTO 560
```

5.

Stage 1: Use the three subroutines quoted in problem 1.

Stage 2: Do all the horizontal lines.

```
  5 REM "8.5"
 10 FOR Y=2 TO 14 STEP 6
 20 LET A=16
 30 LET B=41
 40 GOSUB 400
 50 NEXT Y
 60 FOR Y=20 TO 34 STEP 14
 70 LET A=22
 80 LET B=47
 90 GOSUB 400
100 NEXT Y
110 LET A=26
120 LET B=44
```

```
130 LET Y=35
140 GOSUB 400
142 LET Y=36
144 GOSUB 400
```

Stage 3: Do the two knobs for the drawers.

```
150 PRINT AT 16,14;CHR$ 3
160 PRINT AT 19,14;CHR$ 3
```

Stage 4: Do the four vertical lines.

```
170 FOR X=15 TO 41 STEP 25
180 LET A=3
190 LET B=14
200 GOSUB 500
210 NEXT X
220 LET A=21
230 LET B=34
240 LET X=22
250 GOSUB 500
260 LET A=9
270 LET X=46
280 GOSUB 500
```

Stage 5: Do the three oblique lines. (N.B. Replace line 600 by: 600 FOR Z=0 TO 4.)

```
290 LET B=15
300 FOR A=17 TO 42 STEP 25
310 GOSUB 600
320 NEXT A
330 LET A=42
340 LET B=3
350 GOSUB 600
360 PLOT 47,8
370 GOTO 640
```

6. Two subroutines are needed, one for the three horizontal lines and another for the three oblique lines.

Stage 1: Subroutines:

```
500 FOR Z=A TO B
510 PLOT Z,Y
520 NEXT Z
530 RETURN
600 FOR Z=0 TO 19
610 PLOT A+Z,B+Z
620 NEXT Z
630 RETURN
```

Stage 2: Do the three horizontal lines.

```
  5 REM "8,6"
 10 FOR Y=2 TO 8 STEP 6
 20 LET A=6
 30 LET B=31
 40 GOSUB 500
 50 NEXT Y
 60 LET A=27
 70 LET B=52
 80 LET Y=29
 90 GOSUB 500
```

Stage 3: Do the three oblique lines.

```
100 LET A=7
110 LET B=9
120 GOSUB 600
130 LET A=32
140 GOSUB 600
150 LET B=3
160 GOSUB 600
```

Stage 4: Do the three book ends.

```
170 FOR Y=23 TO 26
180 PLOT 50,Y
190 NEXT Y
200 FOR I=0 TO 25 STEP 25
210 PLOT 5+I,3
220 PLOT 4+I,4
230 PLOT 4+I,5
240 PLOT 4+I,6
250 PLOT 5+I,7
260 NEXT I
270 GOTO 640
```

7. No subroutines are needed here.

Stage 1: The body of the striped hat.

```
  5 REM "8.7"
 10 LET K=-1
 20 FOR Y=9 TO 20
 30 LET K=K+1
 40 FOR X=15+K TO 45-K STEP 2
 50 PLOT X,Y
 60 NEXT X
 70 NEXT Y
```

Stage 2: The bottom and top lines of the hat.

```
 80 FOR X=16 TO 44
 90 PLOT X,8
100 NEXT X
110 FOR X=27 TO 33
120 PLOT X,21
130 NEXT X
```

Stage 3: The top feather.

```
140 PRINT AT 10,14;CHR$ 5
150 FOR I=0 TO 6
160 PLOT 28+I,24+I
170 NEXT I
180 PLOT 35,29
```

Stage 4: The front part of the hat.

```
190 PLOT 14,9
200 FOR I=0 TO 3
210 PLOT 13-I,9+I
220 NEXT I
230 FOR I=0 TO 2
240 PLOT 10+I,13+I
250 NEXT I
```

Stage 5: The back part of the hat.

```
260 PRINT AT 17,23;CHR$ 3
270 PLOT 48,8
280 PLOT 49,7
```

8. We shall use the letter grid to advantage for this. Three subroutines are indicated.

   Stage 1: Subroutines:

```
500 PRINT AT X,Y+2;A$
510 PRINT AT X,Y;A$
520 PRINT AT X+1,Y+1;A$
530 PRINT AT X+2,Y;A$
540 PRINT AT X+2,Y+2;A$
550 RETURN
600 FOR Y=A TO A+3
610 PRINT AT X,Y;A$
620 NEXT Y
630 RETURN
700 FOR X=A TO A+3
710 PRINT AT X,Y;A$
720 NEXT X
730 RETURN
```

   Stage 2: Do all of the horizontal lines.

```
  5 REM "8,8"
 10 LET A$=CHR$ 128
 20 LET A=4
 30 LET X=16
 40 GOSUB 600
 50 LET A=20
 60 LET X=7
 70 GOSUB 600
 80 PRINT AT 16,3;A$
 90 PRINT AT 7,22;A$
100 LET A=5
110 FOR X=8 TO 16 STEP 2
120 LET A=A+2
130 GOSUB 600
140 NEXT X
150 LET A=8
160 FOR X=5 TO 13 STEP 2
170 LET A=A+2
180 GOSUB 600
190 NEXT X
```

   Stage 3: Do the top and bottom squares.

```
200 LET X=4
210 LET Y=6
220 GOSUB 500
230 LET X=16
240 LET Y=20
250 GOSUB 500
```

Stage 4: Do all of the vertical lines.

```
260 LET A=15
270 LET Y=5
280 GOSUB 700
290 PRINT AT 12,5;A$
300 PRINT AT 14,5;A$
310 LET A=3
320 FOR Y=10 TO 18 STEP 2
330 LET A=A+2
340 GOSUB 700
350 NEXT Y
360 LET A=4
370 LET Y=15
380 GOSUB 700
390 LET A=14
400 LET Y=13
410 GOSUB 700
420 LET A=5
430 LET Y=24
440 GOSUB 700
450 PRINT AT 9,24;A$
460 PRINT AT 11,24;A$
470 GOTO 740
```

9.

Stage 1: Use the three subroutines quoted in problem 1.

Stage 2: Do all of the horizontal lines.

```
  5 REM "8,9"
 10 FOR Y=12 TO 30 STEP 3
 20 LET A=17
 30 LET B=44
 40 GOSUB 400
 50 NEXT Y
 60 LET Y=36
 70 LET A=23
 80 LET B=50
 90 GOSUB 400
```

Stage 3: Do the three oblique lines. (N.B. Replace line 600 FOR Z=0 TO 4.)

```
100 LET A=18
110 LET B=31
120 GOSUB 600
130 LET A=45
140 GOSUB 600
150 LET B=13
160 GOSUB 600
```

Stage 4: Do all seven vertical lines.

```
170 FOR I=1 TO 6
180 IF I=1 THEN LET X=17
190 IF I=2 THEN LET X=24
200 IF I=3 THEN LET X=28
210 IF I=4 THEN LET X=33
220 IF I=5 THEN LET X=37
230 IF I=6 THEN LET X=44
240 LET A=12
250 LET B=30
260 GOSUB 500
270 NEXT I
280 LET A=18
290 LET B=36
300 LET X=50
310 GOSUB 500
320 GOTO 640
```

10. (The 16K RAM Pack is not needed here.) No subroutines are needed. For the curved boundaries of the pot the following approximations will be found adequate:

    For the lower arc take $y=0{\cdot}022(x^2-52x+451)+10$
    For the middle arc take $y=0{\cdot}022(x^2-52x+451)+26$
    For the top arc take $y=26-0{\cdot}022(x^2-52x+451)$

    Program as below:

    Stage 1: Do the front of the pot.

```
  5 REM "6.10"
 10 FOR X=11 TO 41
 20 LET Z=INT (0.022*(X*X-52*X+
451))
 30 LET Y1=10+Z
 40 LET Y2=26+Z
 50 FOR Q=Y1 TO Y2
 60 PLOT X,Q
 70 NEXT Q
 80 NEXT X
```

Stage 2: Do the back of the pot.

```
  90 FOR X=11 TO 41
 100 LET Z=-INT (0.022*(X*X-52*X
+451))
 110 LET Y=26+Z
 120 PLOT X,Y
 130 NEXT X
```

Stage 3: Put the handle on.

```
 140 FOR I=1 TO 5
 150 PRINT AT 10,20+I;CHR$ 128
 160 NEXT I
```

11. (The 16K RAM Pack is not needed here.) This design can be done mainly on the letter grid. No subroutines are needed.

Stage 1: Do the body of the vase.

```
  5 REM "8.11"
 10 FOR X=8 TO 21
 20 FOR Y=9 TO 16
 30 PRINT AT X,Y;CHR$ 128
 40 NEXT Y
 50 NEXT X
 60 PRINT AT 21,9;CHR$ 0
 70 PRINT AT 21,16;CHR$ 0
 80 PRINT AT 20,9;CHR$ 133
 90 PRINT AT 20,16;CHR$ 5
```

Stage 2: Do both sides of the vase.

```
 100 FOR K=0 TO 1
 110 FOR X=16+K TO 35-K STEP 19-
2*K
 120 FOR Y=8-2*K TO 19+2*K
 130 PLOT X,Y
 140 NEXT Y
 150 NEXT X
 160 NEXT K
```

12. (The 16K RAM Pack is not needed here.) No subroutines are needed.

Stage 1: The body of the mug.

```
  5 REM "8.12"
 10 FOR X=8 TO 21
 20 FOR Y=10 TO 19
 30 PRINT AT X,Y;CHR$ 128
 40 NEXT Y
 50 NEXT X
 60 PRINT AT 21,10;CHR$ 0
 70 PRINT AT 21,19;CHR$ 0
 80 PRINT AT 20,10;CHR$ 133
 90 PRINT AT 20,19;CHR$ 5
```

Stage 2: The handle of the mug.

```
100 PRINT AT 10,20;CHR$ 131
110 PRINT AT 11,21;CHR$ 134
120 FOR K=0 TO 2
130 PRINT AT 12+K,22;CHR$ 5
140 NEXT K
150 PRINT AT 15,21;CHR$ 6
160 PRINT AT 15,20;CHR$ 131
```

13. (The 16K RAM Pack is not needed here.) No subroutines are needed.

Stage 1: The body of the glass.
A convenient approximation of the curved surface of the glass is

$$y=19+0{\cdot}0216(x^2-62x+637)$$

Program as below:

```
  5 REM "8.13"
 10 FOR X=13 TO 49
 20 LET Z=INT (0.0216*(X*X-62*X
+637))
 30 LET Y=19+Z
 40 FOR Z=Y TO 21
 50 PLOT X,Z
 60 NEXT Z
 70 NEXT X
```

Stage 2: The stem of the glass.

```
 80 FOR Y=0 TO 11
 90 PLOT 31,Y
100 NEXT Y
```

Stage 3: The base of the glass.

```
110 FOR X=24 TO 38
120 PLOT X,0
130 NEXT X
```

14. No subroutines are needed here.

Stage 1: The body of the bottle.

```
  5 REM "8.14"
 10 FOR Y=11 TO 18
 20 FOR X=6 TO 21
 30 PRINT AT X,Y;CHR$ 128
 40 NEXT X
 50 NEXT Y
 60 PRINT AT 21,11;CHR$ 0
 70 PRINT AT 20,11;CHR$ 132
 80 PRINT AT 21,12;CHR$ 132
 90 PRINT AT 21,18;CHR$ 1
100 FOR Y=3 TO 27
110 PLOT 38,Y
120 NEXT Y
```

Stage 2: Linking the body of the bottle to its neck.

```
130 LET K=-1
140 FOR Y=28 TO 33
150 LET K=K+1
160 FOR X=22+K TO 38-K
170 PLOT X,Y
180 NEXT X
190 NEXT Y
```

Stage 3: The neck of the bottle.

```
200 FOR Y=34 TO 41
210 FOR X=28 TO 32
220 PLOT X,Y
230 NEXT X
240 NEXT Y
```

Stage 4: Putting the label on the bottle.

```
250 PRINT AT 14,13;"WINE"
```

15.

Stage 1: Use the three subroutines quoted in problem 1. (Make the subroutines start at 4000, 5000, 6000 respectively.)

Stage 2: Do all the horizontal lines.

```
  5 REM "8.15"
 10 FOR Y=0 TO 23 STEP 23
 20 LET A=18
 30 LET B=45
 50 NEXT Y
 60 LET Y=25
 70 LET A=23
 80 LET B=44
 90 GOSUB 4000
100 LET Y=31
110 LET A=29
120 LET B=50
130 GOSUB 4000
140 FOR Y=33 TO 38 STEP 5
150 LET A=30
160 LET B=53
170 GOSUB 4000
180 NEXT Y
190 PLOT 29,33
200 PLOT 54,33
```

Stage 3: Do the oblique lines. (N.B. Replace line 6000 by 6000 FOR Z=0 TO C.)

```
210 LET A=19
220 LET B=24
230 LET C=9
```

```
250 GOSUB 6000
260 LET A=45
270 GOSUB 6000
280 LET B=1
290 GOSUB 6000
300 LET A=24
310 LET B=26
320 LET C=5
330 GOSUB 6000
340 LET A=45
350 GOSUB 6000
```

Stage 4: Put in the two switches at the top.

```
360 PLOT 37,35
370 PLOT 45,35
```

Stage 5: Do all the vertical lines.

```
380 FOR X=18 TO 45 STEP 27
390 LET A=0
400 LET B=23
410 GOSUB 5000
420 NEXT X
430 FOR X=30 TO 53 STEP 23
440 LET A=34
450 LET B=38
460 GOSUB 5000
470 NEXT X
480 LET X=55
490 LET A=10
500 LET B=32
510 GOSUB 5000
520 GOTO 6040
```

16. No subroutines are needed here.

Stage 1: Do the body of the watering can.

```
 5 REM "8.15"
10 FOR Y=5 TO 17
20 FOR X=6 TO 21
30 PRINT AT X,Y;CHR$ 128
40 NEXT X
50 NEXT Y
60 PRINT AT 21,5;CHR$ 0
70 PRINT AT 21,17;CHR$ 0
```

Stage 2: Put in the spout.

```
 80 FOR K=0 TO 2
 90 LET B=K+6
100 FOR Z=0 TO 13
110 PLOT 36+Z,B+Z
120 NEXT Z
130 NEXT K
140 PLOT 50,20
```

Stage 3: Add the handle.

```
150 PLOT 9,21
160 PLOT 8,22
170 FOR K=0 TO 3
180 PLOT K+4,K+19
190 NEXT K
200 FOR Y=14 TO 18
210 PLOT 4,Y
220 NEXT Y
230 PLOT 5,13
240 PLOT 6,12
250 PLOT 7,11
260 PRINT AT 16,4;CHR$ 3
```

17. No subroutines are needed here.

Stage 1: Front frame of the T.V.

```
  5 REM "8.17"
 10 FOR X=3 TO 18
 20 FOR Y=6 TO 24
 30 PRINT AT X,Y;CHR$ 128
 40 NEXT Y
 50 NEXT X
 60 FOR X=8 TO 16
 70 FOR Y=8 TO 17
 80 PRINT AT X,Y;CHR$ 0
 90 NEXT Y
100 NEXT X
```

Stage 2: Do the control panel.

```
110 FOR Y=27 TO 10 STEP -1
120 FOR X=40 TO 42
130 UNPLOT X,Y
```

```
140 NEXT X
150 NEXT Y
160 PLOT 41,18
170 PLOT 41,14
```

Stage 3: Adjust the top corner.

```
180 PRINT AT 3,6;CHR$ 0
190 PRINT AT 3,7;CHR$ 0
200 PRINT AT 4,6;CHR$ 0
210 FOR K=0 TO 2
220 PRINT AT 3+K,8-K;CHR$ 135
230 NEXT K
```

Stage 4: Adjust the bottom corner.

```
240 PRINT AT 16,23;CHR$ 0
250 PRINT AT 16,24;CHR$ 0
260 PRINT AT 17,24;CHR$ 0
270 FOR K=0 TO 2
280 PRINT AT 16+K,24-K;CHR$ 1
290 NEXT K
```

Stage 5: Fix up the top horizontal line.

```
300 FOR X=18 TO 49
310 PLOT X,38
320 NEXT X
```

Stage 6: Fix up the left vertical line.

```
330 FOR Y=13 TO 38
340 PLOT 50,Y
350 NEXT Y
```

18. Here we simply plot from top to bottom, more or less line by line:

```
  5 REM "8.18"
 10 FOR X=28 TO 35
 20 PLOT X,33
 30 NEXT X
```

```
  40 FOR Z=0 TO 5
  50 PLOT 23+Z,Z+28
  60 NEXT Z
  70 FOR Z=0 TO 5
  80 PLOT 36+Z,33-Z
  90 NEXT Z
 100 PRINT AT 8,11;CHR$ 6;TAB 21
;CHR$ 5
 110 PRINT AT 9,11;CHR$ 5;TAB 16
;CHR$ 131;TAB 21;CHR$ 5
 120 PRINT AT 10,10;CHR$ 6;TAB 1
6;CHR$ 133;TAB 21;CHR$ 5
 130 PLOT 19,21
 140 PRINT AT 11,10;CHR$ 3;CHR$
134
 150 PLOT 42,21
 160 PLOT 41,20
 170 PLOT 32,21
 180 PRINT AT 12,11;CHR$ 133;CHR
$ 131
 190 PLOT 39,18
 200 PLOT 40,19
 210 PLOT 23,17
 220 PLOT 24,16
 230 PRINT AT 13,19;CHR$ 5
 240 PRINT AT 14,12;CHR$ 130;TAB
 19;CHR$ 5
 250 PLOT 26,14
 260 PLOT 27,13
 270 PLOT 28,12
 280 PLOT 35,12
 290 PRINT AT 15,18;CHR$ 131;CHR
$ 3
 300 PLOT 40,13
 310 PRINT AT 16,14;CHR$ 133;CHR
$ 131;CHR$ 3
 320 PLOT 34,11
 330 PLOT 27,9
 340 PLOT 28,9
 350 PLOT 26,25
```

19. No subroutines are needed here.

    Stage 1: The shirt collar.

```
  5 REM "8.19"
 10 PRINT AT 21,9;CHR$ 133
 20 FOR Y=10 TO 22
 30 PRINT AT 21,Y;CHR$ 128
 40 NEXT Y
```

Stage 2: The head (excluding the hair).
Here we assume that the curved boundary is the circle $(x-32)^2+(y-15)^2=338$ and we program as follows:

```
 50 FOR Y=2 TO 28
 60 LET D=INT (SQR (113+30*Y-Y*
Y))
 70 PLOT 32-D,Y
 80 PLOT 32+D,Y
 90 NEXT Y
```

Stage 3: The hair.
Here we assume two parabolic boundaries. For the lower part take

$$y=28+0\cdot0296(x^2-64x+855)$$

and for the upper part assume

$$y=28-0\cdot0296(x^2-64x+855)$$

and program as follows:

```
100 FOR X=19 TO 45
110 LET Y1=INT (0.0295*(X*X-64*
X+855))
120 FOR Y=28+Y1 TO 28-Y1
130 PLOT X,Y
140 NEXT Y
150 NEXT X
```

20. No subroutines are needed here.

Stage 1: The base line of the guitar.

```
  5 REM "6.20"
 10 FOR X=20 TO 34
 20 PLOT X,0
 30 NEXT X
```

Stage 2: The lower part of the guitar. Assume the curve to be part of the circle

$$(x-27)^2+(y-7)^2=98 \text{ for } 1\leqslant y\leqslant 14$$

and program as follows:

```
40 FOR Y=1 TO 14
50 LET D=INT SQR (49+14*Y-Y*Y)
60 PLOT 27-D,Y
70 PLOT 27+D,Y
80 NEXT Y
```

Stage 3: Joining the lower part to the upper part of the guitar.

```
 90 PLOT 21,14
100 PLOT 22,15
110 PRINT AT 13,11;CHR$ 133
120 PRINT AT 14,16;CHR$ 134
130 PRINT AT 13,15;CHR$ 133
```

Stage 4: Do the upper part of the guitar. Assume the curve to be part of the circle

$$(x-27)^2+(y-22)^2=32 \text{ for } 18\leqslant y\leqslant 27$$

and program as follows:

```
140 FOR Y=18 TO 27
150 LET D=INT SQR (44*Y-Y*Y-452
)
160 PLOT 27-D,Y
170 PLOT 27+D,Y
180 NEXT Y
```

Stage 5: The top part of the guitar.

```
190 FOR Y=28 TO 43
200 PLOT 26,Y
210 PLOT 28,Y
220 NEXT Y
230 PLOT 27,43
```

Stage 6: Insert the sound hole.

```
240 FOR Y=17 TO 19
250 FOR X=26 TO 28
260 PLOT X,Y
270 NEXT X
280 NEXT Y
```

21. One subroutine may be used for the two rising beams of the ladder.

Stage 1: Subroutine:

```
600 PRINT AT 21,A;CHR$ 133
610 LET M=0
620 FOR K=1 TO 17 STEP 2
630 LET M=M+1
640 PRINT AT 21-K,M+A;CHR$ 5
650 PRINT AT 20-K,M+A;CHR$ 133
660 NEXT K
670 RETURN
```

Stage 2: The two rising beams.

```
  5 REM "6.21"
 10 LET A=6
 20 GOSUB 600
 30 LET A=15
 40 GOSUB 600
```

Stage 3: The crossbars.

```
  45 LET K=-5
  50 FOR Y=4 TO 34 STEP 10
  60 LET K=K+5
  70 FOR X=20+K TO 33+K
  80 PLOT X,Y
  90 NEXT X
 100 NEXT Y
 110 LET K=-5
 120 FOR Y=9 TO 29 STEP 10
 130 LET K=K+5
 140 FOR X=22+K TO 35+K
 150 PLOT X,Y
 160 NEXT X
 170 NEXT Y
 180 GOTO 680
```

22.

Stage 1: Use the three subroutines quoted in problem 1.

Stage 2: Do all the horizontal lines.

```
  5 REM "8.22"
 10 LET A=17
 20 LET B=40
 30 LET Y=0
 40 GOSUB 400
 50 LET Y=1
 60 GOSUB 400
 70 LET Y=28
 80 GOSUB 400
 90 LET A=28
100 LET B=51
110 LET Y=39
120 GOSUB 400
130 LET A=19
140 LET B=38
150 FOR Y=22 TO 25
160 GOSUB 400
170 NEXT Y
172 PLOT 16,24
174 PLOT 16,23
176 PLOT 39,24
178 PLOT 39,23
```

Stage 3: Do all the vertical lines.

```
180 LET A=0
190 LET B=28
200 LET X=16
210 GOSUB 500
220 LET X=41
230 GOSUB 500
240 LET A=11
250 LET B=39
260 LET X=52
270 GOSUB 500
```

Stage 4: Do the three oblique lines. (N.B. Replace the line 600 by: 600 FOR Z=0 TO 10.)

```
280 LET A=17
290 LET B=29
300 GOSUB 600
```

```
310 LET A=42
320 GOSUB 600
330 LET B=1
340 GOSUB 600
```

Stage 5: Insert the name of the box.

```
390 PRINT AT 15,13;"MAIL"
350 GOTO 640
```

23. Use the subroutine for horizontal straight lines as quoted in problem 1.

Stage 2: The tail end of the plane.

```
  5 REM "8.23"
 10 LET A=29
 20 LET B=35
 30 FOR Y=5 TO 14
 40 GOSUB 400
 50 NEXT Y
 60 PRINT AT 19,15;CHR$ 132
 70 PRINT AT 19,16;CHR$ 128
 80 PLOT 34,5
```

Stage 3: The wing of the plane.

```
 90 LET K=-2
100 FOR Y=15 TO 25
110 LET K=K+2
120 LET A=7+K
130 LET B=57-K
140 GOSUB 400
150 NEXT Y
```

Stage 4: The front end of the plane.

```
160 LET A=29
170 LET B=35
180 FOR Y=26 TO 33
190 GOSUB 400
200 NEXT Y
210 PRINT AT 4,15;CHR$ 129
220 PRINT AT 4,16;CHR$ 128
230 PLOT 34,34
240 GOTO 440
```

24.

Stage 1: Use the subroutine for horizontal straight lines as quoted in problem 1.

Stage 2: Do the cane network of the basket.

```
  4 REM "8.24"
  5 FOR M=0 TO 1
 10 LET K=-1
 20 FOR Y=M+1 TO M+19 STEP 2
 30 LET K=K+1
 40 LET A=18-K
 50 LET B=44+K
 60 GOSUB 400
 70 NEXT Y
 80 NEXT M
```

Stage 3: Do the top half of the basket. Assume a parabolic curve of type

$$y=29+0\cdot01033\ (x^2-62x+477)$$

for the lower top and

$$y=29-0\cdot013033(x^2-62x+477)$$

for the upper top. We now program as follows:

```
  90 FOR X=9 TO 53
 100 LET D=INT (0.01033*(X*X-62*
X+477))
 110 LET Y1=29+D
 120 FOR Y=21 TO Y1
 130 PLOT X,Y
 140 LET Y2=29-D
 150 PLOT X,Y2
 160 NEXT Y
 170 NEXT X
 180 GOTO 440
 400 FOR Z=A TO B STEP 2
 410 PLOT Z,Y
 420 NEXT Z
 430 RETURN
```

25. We need to use the three subroutines of problem 1 plus another one for a wheel.

Stage 1: Extra subroutine:

```
  700 FOR X=A-4 TO A+4
  710 LET D=INT SQR (16-(X-A)*(X-
A))
  720 LET Y1=B-D
  730 LET Y2=B+D
  740 FOR Y=Y1 TO Y2
  750 PLOT X,Y
  760 NEXT Y
  770 NEXT X
  780 RETURN
```

Stage 2: Do all the horizontal lines.

```
    5 REM "8.25"
   10 LET A=8
   20 LET B=38
   30 LET Y=10
   40 GOSUB 400
   50 LET Y=27
   60 GOSUB 400
   70 LET A=16
   80 LET B=47
   90 LET Y=36
  100 GOSUB 400
  110 LET A=44
  120 LET B=61
  130 LET Y=25
  140 GOSUB 400
```

Stage 3: Now do all the vertical lines.

```
  150 LET A=10
  160 LET B=27
  170 LET X=7
  180 GOSUB 500
  190 LET X=39
  200 GOSUB 500
  210 LET A=28
  220 LET B=36
  230 LET X=16
  240 GOSUB 500
  250 LET A=19
  260 LET B=36
  270 LET X=48
  280 GOSUB 500
```

Stage 4: Do the three oblique lines. (Replace line 600 by 600 FOR Z=0 TO 8.)

```
390 LET A=7
400 LET B=27
410 GOSUB 600
420 LET A=39
430 GOSUB 600
440 LET B=15
450 GOSUB 600
```

Stage 5: Put in the two wheels.

```
350 LET A=15
370 LET B=10
380 GOSUB 700
390 LET A=31
395 GOSUB 700
396 GOTO 790
```

26.

Stage 1: Use the subroutines for horizontal straight lines as given in problem 1.

Stage 2: Do the hull of the sailboat.

```
  5 REM "8.26"
 10 FOR M=0 TO 2
 20 LET K=-1
 30 FOR Y=M TO 12+M STEP 3
 40 LET K=K+1
 50 LET A=9-K
 60 LET B=54+K
 70 GOSUB 400
 80 NEXT Y
 90 NEXT M
```

Stage 3: The inscription on the boat.

```
100 PRINT AT 15,4;"T/S 1000"
```

Stage 4: Do the two sails.

```
110 FOR M=0 TO 1
120 LET K=-1
130 FOR Y=16+M TO 40+M STEP 2
140 LET K=K+1
150 LET A=18+K
160 LET B=58-2*K
170 GOSUB 400
180 NEXT Y
190 NEXT M
```

Stage 5: The mast.

```
200 PLOT 31,15
210 PLOT 32,15
220 PRINT AT 0,15;CHR$ 133
225 PRINT AT 0,16;CHR$ 5
```

Stage 6: The inscription on the sail.

```
230 PRINT AT 5,17;"THE"
240 PRINT AT 8,17;"HAPPY"
250 PRINT AT 11,17;"SAILOR"
260 GOTO 440
```

27.

Stage 1: Use the subroutine for vertical lines as quoted in problem 1.

Stage 2: The body of the rocket.

```
  5 REM "8.27"
 10 LET A=1
 20 FOR K=0 TO 2
 30 LET B=31+2*K
 40 LET X=29+K
 50 GOSUB 500
 60 NEXT K
 70 LET B=37
 80 LET X=32
 90 GOSUB 500
```

```
100 FOR K=0 TO 2
110 LET B=31+2*K
120 LET X=35-K
130 GOSUB 500
140 NEXT K
```

Stage 3: The tail end of the rocket.

```
160 LET B=7
170 LET X=27
180 GOSUB 500
190 LET X=37
200 GOSUB 500
210 LET B=8
220 LET X=28
230 GOSUB 500
240 LET X=36
250 GOSUB 500
260 PLOT 29,0
270 PLOT 35,0
```

Stage 4: The label for the rocket.

```
280 PRINT AT 7,16;"M"
290 PRINT AT 8,16;"I"
300 PRINT AT 9,16;"S"
310 PRINT AT 10,16;"S"
320 PRINT AT 11,16;"I"
330 PRINT AT 12,16;"L"
340 PRINT AT 13,16;"E"
350 GOTO 540
```

28. Use the first two subroutines of problem 1 plus the one for a wheel in problem 25.

Stage 1: Do the horizontal lines.

```
  5 REM "8.28"
 10 LET A=4
 20 LET B=52
 30 LET Y=6
 40 GOSUB 400
 50 LET Y=15
 60 GOSUB 400
 70 LET A=20
 80 LET B=39
 90 LET Y=23
100 GOSUB 400
```

Stage 2: Do the vertical lines.

```
110 LET A=5
120 LET B=15
130 LET X=3
140 GOSUB 500
150 LET X=53
160 GOSUB 500
170 LET B=23
180 LET X=29
190 GOSUB 500
200 LET X=40
210 GOSUB 500
220 LET A=16
230 LET B=23
240 LET X=19
250 GOSUB 500
```

Stage 3: Do the windshield.

```
260 FOR Z=0 TO 5
270 PLOT 12+Z,16+Z
280 NEXT Z
```

Stage 4: Do the back window.

```
281 FOR Z=0 TO 6
282 PLOT 41+Z,22-Z
283 NEXT Z
```

Stage 5: The lights and the door handles.

```
284 PRINT AT 15,1;CHR$ 5
285 PRINT AT 15,13;CHR$ 3
286 PRINT AT 15,18;CHR$ 3
```

Stage 6: Put in both wheels.

```
287 LET A=12
288 LET B=6
290 GOSUB 700
300 LET A=42
310 GOSUB 700
320 GOTO 790
```

29. Use the subroutine for horizontal lines of problem 1.

Stage 1: Do the ears.

```
  5 REM "6.29"
 10 LET A=24
 20 LET B=30
 30 FOR Y=36 TO 37
 40 GOSUB 400
 50 NEXT Y
 60 PRINT AT 3,13;CHR$ 130
 70 PRINT AT 2,12;CHR$ 5
 80 PRINT AT 2,11;CHR$ 133
 90 PRINT AT 2,15;CHR$ 128
```

Stage 2: Do the head, mouth and nose. This section is based on part of the circle

$$(x-27)^2+(y-31)^2=36$$

and we program as follows:

```
110 FOR Y=26 TO 36
120 LET D=INT SQR (62*Y-Y*Y-925
)
130 FOR X=27-D TO 27+D
140 PLOT X,Y
150 NEXT X
160 NEXT Y
162 PLOT 21,32
164 PLOT 33,32
166 PRINT AT 7,13;CHR$ 3
168 UNPLOT 28,28
169 UNPLOT 27,30
```

Stage 3: Do its body. This is based on part of the circle

$$(x-27)^2+(y-16)^2=109$$

```
170 FOR Y=5 TO 26
180 LET D=INT SQR (32*Y-Y*Y-147
)
190 FOR X=27-D TO 27+D
200 PLOT X,Y
210 NEXT X
220 NEXT Y
```

Stage 4: Do its tail. This is based on part of the circle

$$(x-37)^2+(y-20)^2=269$$

```
230 FOR X=27 TO 50
240 LET D=INT SQR (74*X-X*X-110
0)
250 PLOT X,20-D
260 NEXT X
```

Stage 5: Do its eyes.

```
270 UNPLOT 25,33
280 UNPLOT 29,33
290 GOTO 440
```

30. Use the subroutine for vertical straight lines for problem 1.

Stage 1: Do the two front columns.

```
  5 REM "8,30"
 10 FOR M=0 TO 28 STEP 28
 20 LET A=4
 30 LET B=33
 40 FOR X=M+6 TO M+14
 50 GOSUB 500
 60 NEXT X
 70 NEXT M
```

Stage 2: Do the central arch. The arch is based on part of the circle with equation

$$(x-24)^2+(y-16)^2=81$$

and we program as follows:

```
 80 FOR X=15 TO 33
 90 LET D=INT SQR (48*X-X*X-495
)
100 FOR Y=16+D TO 33
110 PLOT X,Y
120 NEXT Y
130 NEXT X
135 PLOT 24,24
```

Stage 3: Do the oblique line inside the arch.

```
140 FOR Z=0 TO 15
150 PLOT Z+15,Z+5
160 NEXT Z
```

Stage 4: Do the left side wall.

```
170 LET K=-1
180 FOR X=43 TO 52
185 LET K=K+1
190 LET A=K+5
200 LET B=K+34
210 GOSUB 500
220 NEXT X
```

Stage 5: Do the roof.

```
240 LET K=-1
250 FOR Y=34 TO 43
260 LET K=K+1
270 FOR X=K+7 TO K+41 STEP 2
280 PLOT X,Y
290 NEXT X
300 NEXT Y
310 GOTO 540
```

31.

```
  5 REM "8.31"
 10 FOR Y=0 TO 42 STEP 42
 20 FOR X=2 TO 62 STEP 2
 30 PLOT X,Y
 40 NEXT X
 50 NEXT Y
 60 FOR X=2 TO 62 STEP 60
 70 FOR Y=0 TO 40 STEP 2
 80 PLOT X,Y
 90 NEXT Y
100 NEXT X
110 FOR Y=5 TO 37 STEP 8
120 FOR X=8 TO 56 STEP 8
130 GOSUB 300
140 NEXT X
150 NEXT Y
160 GOTO 350
300 FOR Z=0 TO 4
310 PLOT X+Z-2,Y+Z-2
320 PLOT X+Z-2,Y-Z+2
330 NEXT Z
340 RETURN
```

32. The solution is the same as for problem 31 except that now the subroutine is:

```
295 REM "8.32"
300 PLOT X,Y
310 PLOT X,Y-1
320 PLOT X,Y-2
330 PLOT X-1,Y+1
340 PLOT X+1,Y+1
350 PLOT X-2,Y+2
360 PLOT X+2,Y+2
370 RETURN
```

and line 160 will have to be replaced by 160 GOTO 380.

33. The solution is the same as for problem 31 except that now the subroutine is:

```
295 REM "8.33"
300 FOR A=X-2 TO X+2
310 PLOT A,Y+2
320 PLOT A,Y-2
330 NEXT A
340 FOR Z=0 TO 2
350 PLOT X+Z-1,Y+Z-1
360 NEXT Z
370 RETURN
```

and line 160 will have to be replaced by 160 GOTO 380.

34. The solution is the same as for problem 31 except that now the subroutine is:

```
295 REM "8.34"
300 FOR A=X-1 TO X+1
310 PLOT A,Y-2
320 PLOT A,Y+2
330 NEXT A
340 FOR B=Y-1 TO Y+1
350 PLOT X-2,B
360 PLOT X+2,B
370 NEXT B
380 PLOT X,Y
390 RETURN
```

and line 160 will have to be replaced by 160 GOTO 400.

35. The solution is the same as for problem 31 except that now the subroutine is:

```
295 REM "8.35"
300 FOR A=X-1 TO X+1
310 FOR B=Y-1 TO Y+1
320 PLOT A,B
330 NEXT B
340 NEXT A
350 UNPLOT X,Y
360 PLOT X-2,Y+2
370 PLOT X+2,Y+2
380 PLOT X-2,Y-2
390 PLOT X+2,Y-2
400 RETURN
```

and line 160 will have to be replaced by 160 GOTO 410.

36. The solution is the same as for problem 31 replacing the subroutine by

```
295 REM "8.36"
300 FOR B=Y-2 TO Y+2
310 PLOT X-2,B
320 PLOT X+2,B
330 NEXT B
340 PLOT X-1,Y+1
350 PLOT X,Y
360 PLOT X+1,Y-1
370 RETURN
```

and replacing line 160 by 160 GOTO 380.

37. The solution is the same as for problem 31 but replacing subroutine by

```
295 REM "8.37"
300 PLOT X-2,Y-2
310 FOR B=Y-2 TO Y
320 PLOT X-1,B
330 PLOT X+1,B
340 NEXT B
350 FOR B=Y-2 TO Y+2
360 PLOT X,B
370 NEXT B
380 PLOT X+2,Y-2
390 RETURN
```

and replacing line 160 by 160 GOTO 400.

38. Lines 10 to 120 the same as in problem 31, then program as follows:

```
125 REM 6.38"
130 IF X/15=INT (X/15) THEN GOT
O 160
140 GOSUB 400
150 GOTO 170
160 GOSUB 300
170 NEXT X
180 NEXT Y
190 GOTO 500
300 PLOT X-2,Y-2
310 FOR B=Y-2 TO Y
320 PLOT X-1,B
330 PLOT X+1,B
340 NEXT B
350 FOR B=Y TO Y+2
360 PLOT X,B
370 NEXT B
380 PLOT X+2,Y-2
390 RETURN
400 PLOT X-2,Y+2
410 FOR B=Y TO Y+2
420 PLOT X-1,B
430 PLOT X+1,B
440 NEXT B
450 FOR B=Y-2 TO Y
460 PLOT X,B
470 NEXT B
480 PLOT X+2,Y+2
490 RETURN
```

39. The solution is the same as for problem 38 apart from the two subroutines which are as follows:

```
250 REM "6.39"
300 FOR Z=0 TO 2
310 PLOT X+Z-2,Y+Z
320 NEXT Z
330 FOR Z=0 TO 2
340 PLOT X+Z,Y+Z-2
350 NEXT Z
360 PLOT X-1,Y-1
370 PLOT X+1,Y+1
380 RETURN
400 FOR A=X-2 TO X+2
410 PLOT A,Y-2
420 PLOT A,Y+2
430 NEXT A
440 FOR B=Y-2 TO Y+2
450 PLOT X-2,B
460 PLOT X+2,B
470 NEXT B
480 PLOT X,Y
490 RETURN
```

40. The solution is the same as for problem 38 but the new two subroutines now read:

```
295 REM "8.40"
300 FOR A=X-2 TO X+2 STEP 2
310 FOR B=Y-2 TO Y+2 STEP 2
320 PLOT A,B
330 NEXT B
340 NEXT A
350 RETURN
360 PLOT X-1,Y-1
370 PLOT X+1,Y+1
380 RETURN
400 FOR A=X-1 TO X+1 STEP 2
410 FOR B=Y-2 TO Y+2
420 PLOT A,B
430 NEXT B
440 NEXT A
450 FOR B=Y-1 TO Y+1 STEP 2
460 FOR A=X-2 TO X+2
470 PLOT A,B
480 NEXT A
485 NEXT B
490 RETURN
```

41. The first nine lines do the four bordering edges.

```
  4 REM "8.41"
  5 LET A$=CHR$ 134
 10 FOR X=1 TO 21
 20 PRINT AT X,1;A$
 30 PRINT AT X,30;A$
 40 NEXT X
 50 FOR Y=1 TO 30
 60 PRINT AT 1,Y;A$
 70 PRINT AT 21,Y;A$
 80 NEXT Y
 90 LET B$=CHR$ 7
100 LET C$=CHR$ 131
110 LET D$=CHR$ 3
120 LET E$=CHR$ 132
130 LET F$=CHR$ 133
140 LET G$=CHR$ 23
150 LET H$=CHR$ 128
160 LET I$=CHR$ 5
170 LET J$=CHR$ 6
180 LET K$=CHR$ 61
190 LET L$=CHR$ 52
200 LET M$=CHR$ 130
210 LET N$=CHR$ 129
220 FOR X=5 TO 17 STEP 6
230 FOR Y=5 TO 26 STEP 7
240 GOSUB 600
250 NEXT Y
260 NEXT X
270 GOTO 660
```

```
 600 PRINT AT X-2,Y-2;B$;C$;D$;C
$;E$
 610 PRINT AT X-1,Y-2;F$;G$;H$;G
$;I$
 620 PRINT AT X,Y-2;I$;J$;K$;A$;
F$
 630 PRINT AT X+1,Y-2;F$;G$;H$;G
$;I$
 640 PRINT AT X+2,Y-2;M$;D$;C$;D
$;N$
 650 RETURN
```

42. The solution is as in problem 41 but now the subroutine is:

```
 595 REM "8.42"
 600 PRINT AT X-2,Y-2;J$;A$;J$;A
$;A$
 610 PRINT AT X-1,Y-2;A$;K$;G$;K
$;J$
 620 PRINT AT X,Y-2;I$;G$;H$;G$;
F$
 630 PRINT AT X+1,Y-2;J$;K$;G$;K
$;J$
 640 PRINT AT X+2,Y-2;A$;J$;A$;J
$;A$
 650 RETURN
```

43. The solution is as in problem 41 with the following subroutine:

```
 595 REM "8.43"
 600 PRINT AT X-2,Y-2;G$;D$;K$;D
$;G$
 610 PRINT AT X-1,Y-2;M$;G$;H$;G
$;N$
 620 PRINT AT X,Y-2;K$;H$;H$;H$;
K$
 630 PRINT AT X+1,Y-2;B$;G$;H$;G
$;E$
 640 PRINT AT X+2,Y-2;G$;C$;K$;C
$;G$
 650 RETURN
```

44. The solution is as in problem 41 with the following subroutine:

```
 595 REM "8.44"
 600 PRINT AT X-2,Y-2;B$;D$;D$;D
$;E$
 610 PRINT AT X-1,Y-2;I$;K$;L$;K
$;F$
 620 PRINT AT X,Y-2;I$;L$;G$;L$;
F$
```

```
 630 PRINT AT X+1,Y-2;I$;K$;L$;K
$;F$
 640 PRINT AT X+2,Y-2;M$;C$;C$;C
$;N$
 650 RETURN
```

45. The solution is as in problem 41 with the following subroutine:

```
 595 REM "8.45"
 600 PRINT AT X-2,Y-2;L$;G$;H$;G
$;L$
 610 PRINT AT X-1,Y-2;G$;K$;H$;K
$;G$
 620 PRINT AT X,Y-2;H$;H$;H$;H$;
H$
 630 PRINT AT X+1,Y-2;G$;K$;H$;K
$;G$
 640 PRINT AT X+2,Y-2;L$;G$;H$;G
$;L$
 650 RETURN
```

46. As in problem 41 with the new subroutine:

```
 595 REM "8.46"
 600 PRINT AT X-2,Y-2;A$;A$;A$;A
$;A$
 610 PRINT AT X-1,Y-2;A$;K$;G$;K
$;A$
 620 PRINT AT X,Y-2;A$;G$;L$;G$;
A$
 630 PRINT AT X+1,Y-2;A$;K$;G$;K
$;A$
 640 PRINT AT X+2,Y-2;A$;A$;A$;A
$;A$
 650 RETURN
```

47. As in problem 41 with the new subroutine:

```
 595 REM "8.47"
 600 PRINT AT X-2,Y-2;L$;L$;L$;L
$;L$
 610 PRINT AT X-1,Y-2;L$;K$;K$;K
$;L$
 620 PRINT AT X,Y-2;L$;K$;G$;K$;
L$
 630 PRINT AT X+1,Y-2;L$;K$;K$;K
$;L$
 640 PRINT AT X+2,Y-2;L$;L$;L$;L
$;L$
 650 RETURN
```

48. As in problem 41 but now the subroutine reads:

```
 595 REM "8.48"
 600 PRINT AT X-1,Y-2;K$;K$;K$;K
$;K$
 610 PRINT AT X-1,Y-2;K$;G$;H$;G
$;K$
 620 PRINT AT X,Y-2;K$;A$;L$;J$;
K$
 630 PRINT AT X+1,Y-2;K$;G$;H$;G
$;K$
 640 PRINT AT X+2,Y-2;K$;K$;K$;K
$;K$
 650 RETURN
```

49. As in problem 41 with the new subroutine:

```
 595 REM "8.49"
 600 PRINT AT X-2,Y-2;G$;G$;G$;G
$;G$
 610 PRINT AT X-1,Y-2;G$;K$;L$;K
$;G$
 620 PRINT AT X,Y-2;G$;L$;H$;L$;
G$
 630 PRINT AT X+1,Y-2;G$;K$;L$;K
$;G$
 640 PRINT AT X+2,Y-2;G$;G$;G$;G
$;G$
 650 RETURN
```

50. As in problem 41 with the new subroutine:

```
 595 REM "8.50"
 600 PRINT AT X-2,Y-2;G$;K$;H$;K
$;G$
 610 PRINT AT X-1,Y-2;K$;G$;L$;G
$;K$
 620 PRINT AT X,Y-2;H$;L$;G$;L$;
H$
 630 PRINT AT X+1,Y-2;K$;G$;L$;G
$;K$
 640 PRINT AT X+2,Y-2;G$;K$;H$;K
$;G$
 650 RETURN
```

Chapter 9

1. Add the following modifications to problem 5, Chapter 8:

```
635 REM "9.1"
540 FOR K=1 TO 20
650 FOR X=27 TO 43
660 UNPLOT X,35
670 NEXT X
680 FOR X=27 TO 43
690 PLOT X,35
700 NEXT X
710 NEXT K
```

2. Modify problem 18 of Chapter 8 as follows:

```
355 REM "9.2"
360 FOR K=1 TO 20
370 UNPLOT 26,25
380 GOSUB 500
400 PLOT 26,25
420 GOSUB 500
430 NEXT K
440 GOTO 530
500 FOR R=1 TO 15
510 NEXT R
520 RETURN
```

3. Modify problem 18 of Chapter 8 as follows:

```
356 REM "9.3"
360 FOR K=1 TO 20
370 UNPLOT 26,25
380 PLOT 26,19
385 GOSUB 500
390 GOSUB 500
400 PLOT 26,25
410 UNPLOT 26,19
420 GOSUB 500
430 NEXT K
440 GOTO 530
500 FOR R=1 TO 25
510 NEXT R
520 RETURN
```

4. Modify illustration 3 of Chapter 8 as follows:

```
265 REM "9.4"
270 FOR M=1 TO 20
280 FOR K=1 TO 6
290 PRINT AT 7,13+K;CHR$ 23
300 PRINT AT 4,16+K;CHR$ 23
310 PRINT AT 7,16+K;CHR$ 23
320 PRINT AT 6,18+K;CHR$ 23
330 PRINT AT 5,20+K;CHR$ 23
340 PRINT AT 4,23+K;CHR$ 23
350 GOSUB 600
360 PRINT AT 7,13+K;CHR$ 0
370 PRINT AT 4,16+K;CHR$ 0
380 PRINT AT 7,16+K;CHR$ 0
390 PRINT AT 6,18+K;CHR$ 0
400 PRINT AT 5,20+K;CHR$ 0
410 PRINT AT 4,23+K;CHR$ 0
420 GOSUB 600
430 NEXT K
440 NEXT M
450 GOTO 630
500 FOR Z=A TO B STEP C
510 PLOT Z,Y
520 NEXT Z
530 RETURN
600 FOR R=1 TO 50
610 NEXT R
620 RETURN
```

5. Modify problem 7 of Chapter 8 as follows:

```
285 REM "9.5"
290 FOR M=1 TO 20
300 FOR I=0 TO 6
310 UNPLOT 28+I,24+I
320 NEXT I
330 UNPLOT 35,29
340 GOSUB 600
350 PRINT AT 9,14;CHR$ 131
360 PRINT AT 9,15;CHR$ 3
370 PRINT AT 8,16;CHR$ 131
380 PRINT AT 8,17;CHR$ 3
390 GOSUB 600
400 PRINT AT 9,14;CHR$ 0
410 PRINT AT 9,15;CHR$ 0
420 PRINT AT 8,16;CHR$ 0
430 PRINT AT 8,17;CHR$ 0
440 GOSUB 600
450 FOR I=0 TO 6
460 PLOT 28+I,24+I
470 NEXT I
480 PLOT 35,29
490 GOSUB 600
500 NEXT M
510 GOTO 630
600 FOR R=1 TO 10
610 NEXT R
620 RETURN
```

6. Modify problem 16 of Chapter 8 as follows:

```
 265 REM "9.6"
 270 FOR K=1 TO 20
 280 FOR X=50 TO 62 STEP 2
 290 LET D=INT (0.222*(X-50)*(X-
56))
 300 PLOT X,21-D
 310 NEXT X
 320 GOSUB 600
 330 FOR X=50 TO 62 STEP 2
 340 LET D=INT (0.222*(X-50)*(X-
56))
 350 UNPLOT X,21-D
 360 NEXT X
 370 GOSUB 600
 380 NEXT K
 390 GOTO 630
 600 FOR R=1 TO 20
 610 NEXT R
 620 RETURN
```

7. Modify problem 28 of Chapter 8 as follows:

```
 785 REM "9.7"
 790 FOR K=1 TO 20
 800 FOR X=63 TO 0 STEP -3
 810 PLOT X,5
 820 NEXT X
 830 FOR X=63 TO 0 STEP -3
 840 UNPLOT X,5
 850 NEXT X
 860 NEXT K
```

8. Modify problem 25 of Chapter 8 as follows:

```
 785 REM "9.8"
 790 FOR K=1 TO 20
 800 FOR X=0 TO 62 STEP 3
 810 PLOT X,1
 820 NEXT X
 830 FOR X=0 TO 62 STEP 3
 840 UNPLOT X,1
 850 NEXT X
 860 NEXT K
```

9.

```
  5 REM "9.9"
 10 FOR K=1 TO 20
 20 FOR T=0 TO 2*PI STEP PI/15
 30 LET A=32+20*COS T
 40 LET B=22+20*SIN T
 50 PLOT A,B
 60 GOSUB 200
 70 UNPLOT A,B
 80 NEXT T
 90 NEXT K
100 GOTO 230
200 FOR R=1 TO 20
210 NEXT R
220 RETURN
```

10. This is based on 13 parallel vertical lines going from left to right.

```
 5 REM "9.10"
10 FOR K=1 TO 40
20 FOR X=0 TO 60 STEP 5
30 FOR Y=43 TO 0 STEP -4
35 PLOT X,Y
40 NEXT Y
50 NEXT X
60 CLS
70 NEXT K
```

11.

```
  5 REM "9.11"
 10 FOR K=1 TO 20
 20 FOR I=1 TO 4
 30 FOR X=4 TO 17
 40 FOR Y=5 TO 16
 50 IF I=1 THEN GOSUB 300
 60 IF I=2 THEN GOSUB 400
 70 IF I=3 THEN GOSUB 500
 75 IF I=4 THEN GOSUB 600
 80 NEXT Y
 90 NEXT X
100 CLS
110 NEXT I
120 NEXT K
130 GOTO 620
300 PRINT AT X,Y;CHR$ 23
310 RETURN
400 PRINT AT X,Y;CHR$ 134
410 RETURN
500 PRINT AT X,Y;CHR$ 13
510 RETURN
600 PRINT AT X,Y;CHR$ 21
610 RETURN
```

12.

```
  5 REM "9.12"
 10 LET A$=CHR$ 23
 20 LET B$=CHR$ 21
 30 LET C$=CHR$ 56
 40 LET D$=CHR$ 63
 50 LET E$=CHR$ 46
 60 LET F$=CHR$ 62
 70 LET G$=CHR$ 57
 80 LET H$=CHR$ 61
 90 LET I$=CHR$ 52
100 FOR K=1 TO 20
110 FOR I=1 TO 2
140 IF I=1 THEN GOSUB 300
150 IF I=2 THEN GOSUB 400
160 CLS
190 NEXT I
200 NEXT K
210 GOTO 450
300 PRINT AT 6,10;A$;B$;A$;B$;A
$;B$;A$
310 PRINT AT 7,10;B$;C$;D$;E$;D
$;C$;B$
320 PRINT AT 8,10;A$;F$;G$;E$;G
$;F$;A$
330 PRINT AT 9,10;B$;H$;E$;G$;E
$;H$;B$
340 PRINT AT 10,10;A$;F$;G$;E$;
D$;F$;A$
350 PRINT AT 11,10;B$;C$;D$;E$;
D$;C$;B$
360 PRINT AT 12,10;A$;B$;A$;B$;
A$;B$;A$
365 PAUSE 25
370 RETURN
400 PRINT AT 6,10;H$;H$;H$;H$;H
$;H$;H$
410 PRINT AT 7,10;H$;I$;I$;I$;I
$;I$;H$
420 PRINT AT 8,10;H$;I$;A$;A$;A
$;I$;H$
430 PRINT AT 9,10;H$;I$;A$;A$;A
$;I$;H$
440 PRINT AT 10,10;H$;I$;A$;A$;
A$;I$;H$
450 PRINT AT 11,10;H$;I$;I$;I$;
I$;I$;H$
460 PRINT AT 12,10;H$;H$;H$;H$;
H$;H$;H$
465 PAUSE 25
470 RETURN
```

13.

```
  5 REM "9.13"
 10 FOR K=1 TO 20
 20 FOR I=1 TO 4
 30 LET M=0
 40 FOR X=5 TO 12
 50 LET M=M+1
 55 FOR Y=5 TO 11 STEP 2
 60 IF I=1 THEN GOSUB 300
 70 IF I=2 THEN GOSUB 400
 80 IF I=3 THEN GOSUB 500
 90 IF I=4 THEN GOSUB 600
100 NEXT Y
110 NEXT X
120 CLS
130 NEXT I
140 NEXT K
150 GOTO 650
300 IF M/2=INT (M/2) THEN GOTO
330
310 PRINT AT X,Y;CHR$ 21;CHR$ 2
3
320 GOTO 340
330 PRINT AT X,Y;CHR$ 23;CHR$ 2
1
335 IF X=12 AND Y=11 THEN PAUSE
25
340 RETURN
400 IF M/2=INT (M/2) THEN GOTO
430
410 PRINT AT X,Y;CHR$ 52;CHR$ 6
1
420 GOTO 440
430 PRINT AT X,Y;CHR$ 61;CHR$ 5
2
435 IF X=12 AND Y=11 THEN PAUSE
25
440 RETURN
500 IF M/2=INT (M/2) THEN GOTO
530
510 PRINT AT X,Y;CHR$ 50;CHR$ 6
0
520 GOTO 540
530 PRINT AT X,Y;CHR$ 60;CHR$ 5
0
535 IF X=12 AND Y=11 THEN PAUSE
25
540 RETURN
600 IF M/2=INT (M/2) THEN GOTO
630
610 PRINT AT X,Y;CHR$ 12;CHR$ 1
3
620 GOTO 640
630 PRINT AT X,Y;CHR$ 13;CHR$ 1
2
635 IF X=12 AND Y=11 THEN PAUSE
25
640 RETURN
```

14.

```
  5 REM "9.14"
 10 LET A$=CHR$ 134
 20 LET B$=CHR$ 61
 30 LET C$=CHR$ 50
 40 LET D$=CHR$ 23
 50 LET E$=CHR$ 60
 60 LET F$=CHR$ 21
 70 LET G$=CHR$ 52
 80 LET H$=CHR$ 13
 90 LET I$=CHR$ 6
100 LET J$=CHR$ 63
110 FOR K=1 TO 20
120 FOR I=1 TO 4
130 IF I=1 THEN GOSUB 300
140 IF I=2 THEN GOSUB 400
150 IF I=3 THEN GOSUB 500
160 IF I=4 THEN GOSUB 600
170 CLS
180 NEXT I
190 NEXT K
200 GOTO 660
300 PRINT AT 6,10;A$;A$;A$;A$;A
$;A$;A$
310 PRINT AT 7,10;A$;B$;C$;D$;C
$;B$;A$
320 PRINT AT 8,10;A$;E$;F$;D$;F
$;E$;A$
330 PRINT AT 9,10;A$;D$;G$;D$;G
$;D$;A$
340 PRINT AT 10,10;A$;E$;F$;D$;
F$;E$;A$
350 PRINT AT 11,10;A$;B$;C$;D$;
C$;B$;A$
360 PRINT AT 12,10;A$;A$;A$;A$;
A$;A$;A$
365 PAUSE 25
370 RETURN
400 PRINT AT 6,10;H$;H$;H$;H$;H
$;H$;H$
410 PRINT AT 7,10;H$;B$;D$;G$;D
$;B$;H$
420 PRINT AT 8,10;H$;D$;B$;F$;B
$;D$;H$
430 PRINT AT 9,10;H$;G$;F$;B$;F
$;G$;H$
440 PRINT AT 10,10;H$;D$;B$;F$;
B$;D$;H$
450 PRINT AT 11,10;H$;B$;D$;G$;
D$;E$;H$
460 PRINT AT 12,10;H$;H$;H$;H$;
H$;H$;H$
465 PAUSE 25
470 RETURN
500 PRINT AT 6,10;I$;A$;I$;A$;I
$;A$;I$
510 PRINT AT 7,10;A$;G$;F$;C$;F
$;G$;A$
520 PRINT AT 8,10;I$;F$;G$;D$;G
$;F$;I$
```

```
 530 PRINT AT 9,10;A$;B$;D$;G$;D
$;B$;A$
 540 PRINT AT 10,10;I$;F$;G$;D$;
G$;F$;I$
 550 PRINT AT 11,10;A$;G$;F$;E$;
F$;G$;A$
 560 PRINT AT 12,10;I$;A$;I$;A$;
I$;A$;I$
 565 PAUSE 25
 570 RETURN
 600 PRINT AT 6,10;J$;J$;J$;J$;J
$;J$;J$
 610 PRINT AT 7,10;J$;F$;E$;D$;E
$;F$;J$
 620 PRINT AT 8,10;J$;E$;F$;D$;F
$;E$;J$
 630 PRINT AT 9,10;J$;D$;D$;F$;D
$;D$;J$
 640 PRINT AT 10,10;J$;C$;F$;D$;
F$;C$;J$
 650 PRINT AT 11,10;J$;F$;C$;D$;
C$;F$;J$
 660 PRINT AT 12,10;J$;J$;J$;J$;
J$;J$;J$
 665 PAUSE 25
 670 RETURN
```

15. Let us assume that the kaleidoscope is a circle with equation

$$(x-11)^2+(y-16)^2=49$$

in the letter grid.
We then program as follows:

```
  5 REM "9.15"
 20 FOR K=1 TO 20
 30 FOR I=1 TO 4
 35 FOR X=4 TO 18
 37 LET D=INT SQR (22*X-X*X-72)
 40 FOR Y=16-D TO 16+D
 50 IF I=1 THEN GOSUB 300
 60 IF I=2 THEN GOSUB 400
 70 IF I=3 THEN GOSUB 500
 75 IF I=4 THEN GOSUB 600
 80 NEXT Y
 90 NEXT X
120 CLS
130 NEXT I
140 NEXT K
150 GOTO 620
```

The four subroutines are the same as before.

16. The next 19 lines are as before. Then program:

```
 170 REM "9.16"
 180 CLS
 190 NEXT I
 200 NEXT K
 210 GOTO 480
 300 PRINT AT 6,11;B$;A$;B$;A$;B
$
 310 PRINT AT 7,10;B$;C$;D$;E$;D
$;C$;B$
 320 PRINT AT 8,10;A$;F$;G$;E$;G
$;F$;A$
 330 PRINT AT 9,10;B$;H$;E$;G$;E
$;H$;B$
 340 PRINT AT 10,10;A$;F$;G$;E$;
G$;F$;A$
 350 PRINT AT 11,10;B$;C$;D$;E$;
D$;C$;B$
 360 PRINT AT 12,11;B$;A$;B$;A$;
B$
 370 RETURN
 400 PRINT AT 6,11;H$;H$;H$;H$;H
$
 410 PRINT AT 7,10;H$;I$;I$;I$;I
$;I$;H$
 420 PRINT AT 8,10;H$;I$;A$;A$;A
$;I$;H$
 430 PRINT AT 9,10;H$;I$;A$;A$;A
$;I$;H$
 440 PRINT AT 10,10;H$;I$;A$;A$;
A$;I$;H$
 450 PRINT AT 11,10;H$;I$;I$;I$;
I$;I$;H$
 460 PRINT AT 12,11;H$;H$;H$;H$;
H$
 470 RETURN
```

17.
```
  5 REM "9.17"
  6 FAST
```

The next six lines are as in problem 13 above, then

```
 60 IF I=1 THEN GOTO 100
 70 IF I=2 THEN GOTO 130
 80 IF I=3 THEN GOTO 160
 90 IF I=4 THEN GOTO 190
100 LET P=21
110 LET Q=23
120 GOTO 210
130 LET P=52
140 LET Q=61
150 GOTO 210
160 LET P=50
170 LET Q=60
180 GOTO 210
```

```
 190 LET P=12
 200 LET Q=13
 210 GOSUB 400
 220 NEXT Y
 230 NEXT X
 240 PAUSE 120
 250 POKE 16437,255
 260 CLS
 270 NEXT I
 280 NEXT K
 290 GOTO 530
 400 IF M=1 THEN GOTO 470
 405 IF M=3 THEN GOTO 472
 410 IF M=2 THEN GOTO 500
 415 IF M=7 THEN GOTO 502
 420 IF M/2=INT (M/2) THEN GOTO
450
 430 PRINT AT X,Y;CHR$ Q;CHR$ P
 440 GOTO 520
 450 PRINT AT X,Y;CHR$ P;CHR$ Q
 460 GOTO 520
 470 PRINT AT X,Y+2;CHR$ Q;CHR$
P
 471 GOTO 480
 472 PRINT AT X,Y+2;CHR$ P;CHR$
Q
 480 IF Y=9 OR Y=11 THEN PRINT A
T X,Y+2;CHR$ Q;CHR$ Q
 490 GOTO 520
 500 PRINT AT X,Y+1;CHR$ Q;CHR$
P
 501 GOTO 510
 502 PRINT AT X,Y+1;CHR$ P;CHR$
Q
 510 IF Y=11 THEN PRINT AT X,12;
CHR$ Q;CHR$ Q
 520 RETURN
```

18. The next 16 lines are as in problem 14. Now program:

```
 185 REM "9.18"
 190 CLS
 200 NEXT I
 210 NEXT K
 220 GOTO 680
 300 PRINT AT 6,11;A$;A$;A$;A$;A
$
 310 PRINT AT 7,10;A$;B$;C$;D$;C
$;B$;A$
 320 PRINT AT 8,10;A$;E$;F$;D$;F
$;E$;A$
 330 PRINT AT 9,10;A$;D$;G$;D$;G
$;D$;A$
 340 PRINT AT 10,10;A$;E$;F$;D$;
F$;E$;A$
 350 PRINT AT 11,10;A$;B$;C$;D$;
C$;B$;A$
 360 PRINT AT 12,11;A$;A$;A$;A$;
A$
```

```
 370 RETURN
 400 PRINT AT 6,11;H$;H$;H$;H$;H
$
 410 PRINT AT 7,10;H$;B$;D$;G$;D
$;B$;H$
 420 PRINT AT 8,10;H$;D$;B$;F$;B
$;D$;H$
 430 PRINT AT 9,10;H$;G$;F$;B$;F
$;G$;H$
 440 PRINT AT 10,10;H$;D$;B$;F$;
B$;D$;H$
 450 PRINT AT 11,10;H$;B$;D$;G$;
D$;B$;H$
 460 PRINT AT 12,11;H$;H$;H$;H$;
H$
 470 RETURN
 500 PRINT AT 6,11;A$;I$;A$;I$;A
$
 510 PRINT AT 7,10;A$;G$;F$;C$;F
$;G$;A$
 520 PRINT AT 8,10;I$;F$;G$;D$;G
$;F$;I$
 530 PRINT AT 9,10;A$;B$;D$;G$;D
$;B$;A$
 540 PRINT AT 10,10;I$;F$;G$;D$;
G$;F$;I$
 550 PRINT AT 11,10;A$;G$;F$;E$;
F$;G$;A$
 560 PRINT AT 12,11;A$;I$;A$;I$;
A$
 570 RETURN
 600 PRINT AT 6,11;J$;J$;J$;J$;J
$
 610 PRINT AT 7,10;J$;F$;E$;D$;E
$;F$;J$
 620 PRINT AT 8,10;J$;E$;F$;D$;F
$;E$;J$
 630 PRINT AT 9,10;J$;D$;D$;F$;D
$;D$;J$
 640 PRINT AT 10,10;J$;C$;F$;D$;
F$;C$;J$
 650 PRINT AT 11,10;J$;F$;C$;D$;
C$;F$;J$
 660 PRINT AT 12,11;J$;J$;J$;J$;
J$
 670 RETURN
```

19.

```
  5 REM "9.19"
 20 FOR K=1 TO 3
 30 FOR N=38 TO 63
 35 FOR X=4 TO 18
 37 LET D=INT SQR (22*X-X*X-72)
 40 FOR Y=16-D TO 16+D
 50 GOSUB 500
 60 NEXT Y
 70 NEXT X
```

```
100 CLS
110 NEXT N
120 NEXT K
130 GOTO 520
500 PRINT AT X,Y;CHR$ N
510 RETURN
```

20.

```
  5 REM "9.20"
 20 LET A$=CHR$ 3
 30 LET B$=CHR$ 7
 40 LET C$=CHR$ 23
 50 LET D$=CHR$ 81
 60 LET E$=CHR$ 62
 70 LET F$=CHR$ 52
 80 LET G$=CHR$ 5
 90 LET H$=CHR$ 133
100 LET I$=CHR$ 132
110 LET J$=CHR$ 130
120 LET K$=CHR$ 129
130 LET L$=CHR$ 21
140 LET M$=CHR$ 50
150 LET N$=CHR$ 63
155 LET O$=CHR$ 131
160 FOR K=1 TO 20
170 FOR I=1 TO 4
180 IF I=1 THEN GOSUB 300
190 IF I=2 THEN GOSUB 400
200 IF I=3 THEN GOSUB 500
210 IF I=4 THEN GOSUB 600
240 CLS
250 NEXT I
260 NEXT K
270 GOTO 680
300 PRINT AT 6,11;A$;A$;A$;A$
310 PRINT AT 7,10;B$;C$;D$;D$;C
$;I$
320 PRINT AT 8,9;G$;E$;F$;L$;L$
;F$;E$;H$
330 PRINT AT 9,9;G$;C$;L$;F$;F$
;L$;C$;H$
340 PRINT AT 10,9;G$;C$;L$;F$;F
$;L$;C$;H$
350 PRINT AT 11,9;G$;E$;F$;L$;L
$;F$;E$;H$
360 PRINT AT 12,10;J$;C$;D$;D$;
C$;K$
370 PRINT AT 13,11;O$;O$;O$;O$
380 RETURN
400 PRINT AT 6,11;L$;L$;L$;L$
410 PRINT AT 7,10;F$;M$;M$;M$;M
$;F$
420 PRINT AT 8,9;L$;M$;F$;D$;D$
;F$;M$;L$
430 PRINT AT 9,9;L$;M$;D$;C$;C$
;D$;M$;L$
440 PRINT AT 10,9;L$;M$;D$;C$;C
$;D$;M$;L$
```

```
 450 PRINT AT 11,9;L$;M$;F$;D$;D
$;F$;M$;L$
 460 PRINT AT 12,10;F$;M$;M$;M$;
M$;F$
 470 PRINT AT 13,11;L$;L$;L$;L$
 480 RETURN
 500 PRINT AT 6,11;C$;C$;C$;C$
 510 PRINT AT 7,10;C$;L$;L$;L$;L
$;C$
 520 PRINT AT 8,9;C$;L$;D$;F$;F$
;D$;L$;C$
 530 PRINT AT 9,9;C$;L$;F$;N$;N$
;F$;L$;C$
 540 PRINT AT 10,9;C$;L$;F$;N$;N
$;F$;L$;C$
 550 PRINT AT 11,9;C$;L$;D$;F$;F
$;D$;L$;C$
 560 PRINT AT 12,10;C$;L$;L$;L$;
L$;C$
 570 PRINT AT 13,11;C$;C$;C$;C$
 580 RETURN
 600 PRINT AT 6,11;D$;D$;D$;D$
 610 PRINT AT 7,10;D$;L$;L$;L$;L
$;D$
 620 PRINT AT 8,9;D$;C$;E$;F$;F$
;E$;C$;D$
 630 PRINT AT 9,9;D$;C$;F$;D$;D$
;F$;C$;D$
 640 PRINT AT 10,9;D$;C$;F$;D$;D
$;F$;C$;D$
 650 PRINT AT 11,9;D$;C$;E$;F$;F
$;E$;C$;D$
 660 PRINT AT 12,10;D$;L$;L$;L$;
L$;D$
 670 PRINT AT 13,11;D$;D$;D$;D$
 680 RETURN
```

21.

```
  5 REM "9.21"
 10 FAST
 20 FOR K=1 TO 10
 30 FOR A=0 TO 23
 40 GOSUB 500
 50 PAUSE 60
 60 POKE 16437,255
 70 CLS
 80 NEXT A
 90 NEXT K
100 GOTO 590
500 FOR Y=A+1 TO A+7 STEP 6
510 FOR X=15 TO 20
520 PRINT AT X,Y;CHR$ 128
530 NEXT X
540 NEXT Y
550 FOR Y=A TO A+6
560 PRINT AT 14,Y;CHR$ 128
570 NEXT Y
580 RETURN
```

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